The Core of the Party System*

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Abstract

A core party is undefeated in pair-wise majority preference comparisons against all other political parties. We use survey self-reported voter preference data and moment inequalities testing methods to identify potential core and Condorcet winner parties. We carry out tests for 1,176 parties over 196 election surveys. We cannot reject the hypothesis of a core party in nearly half the tests, while we cannot reject the hypothesis that a party is a Condorcet winner in only about 7.06% of tests. We construct both discrete and continuous measures of core and Condorcet parties and probe the face-validity of these measures using party-specific observables. On the subset of multiparty non-presidential democracies, these parties are between 20% and 30% more likely to participate in a post-election government, and between 10% (for core) and 40% (for Condorcet parties) more likely to control the premiership, even when controlling for common predictors of holding such office.

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1 Introduction

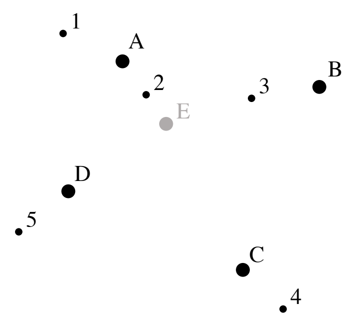
In most modern representative democracies voters are first presented with a choice primarily structured over a set of political parties and then policy-making proceeds after the election through preference aggregation among the subset of these parties that earn representation in the legislature. It is often hard to identify beforehand how much leverage different parties will end up exercising in post-election policy-making. We develop an empirical method to identify a key subset of political parties that are likely to play a lead part in this type of party-centered democratic representation. The parties that we flag out for this protagonist role are in the *core* of the party system in the social choice sense: A core party is undefeated in pairwise majority preference comparisons against all other parties. Relatedly, a Condorcet winner (henceforth a Condorcet party) is a party that defeats every other party in pairwise majority comparisons. If such parties exist, there are both eminently germane normative reasons and (as we elaborate later on in this introduction) positive theories supporting the expectation that they are more likely to govern. Our goal in this paper is three-fold: First, to develop a method to identify possible core and Condorcet parties. Second, to evaluate the face-validity of the measurement method by showing that it identifies parties that correlate consistently with theoretically relevant observed attributes, while still being distinct from these attributes. Third, to take a first step at evaluating the usefulness of the resulting measures to empirically assess normative expectations about core and Condorcet parties and/or as empirical predictors of government participation.

For the first step in this research agenda, we propose a test of the hypothesis that a party is a core party as well as the hypothesis that it is a Condorcet party using tests of moment inequalities. These tests can be implemented using survey questions routinely incorporated in most national election studies, that is, they can identify possible core and Condorcet parties using data that are either readily available or can be obtained even before an election. Readers familiar with the broad lessons of social choice theory may hasten to object that we are pursuing a fool's errand because the conditions for the existence of a majority core, let alone a Condorcet winner, are almost surely not met when the electorates' preferences are even mildly diverse (Plott 1967). The key qualification here is that we apply the idea of the core on the finite set of existing parties, not on the universe of possible parties that the relevant party attribute space encompasses. Indeed any of our core parties, if construed as a point (a policy or platform or a more encompassing bundle[1]) located on some continuous attribute space, may well be majority-defeated by some other point in that continuum. In our nomenclature, that party is a core party as long as it is not defeated by any one of a finite number of *existing* parties. Within that finite set, a core party and even a Condorcet winner may exist in a robust way, independent of the dimensionality of the ambient space of party attributes. These ideas are illustrated in Figure 1 in a two-dimensional space with four parties (A, B, C, and D) and five voters. The Plott symmetry conditions are not met in this electorate and therefore the core in the entire two-dimensional space is empty. Nevertheless, party A is a Condorcet winner according to our definition as it is strictly majority preferred over each of parties B, C, and D. Furthermore, this property is robust to small changes in the location of parties A, B, C, and D (or that of the voters).

Even within the confines of our definition, determining whether a party belongs in the core requires detailed information about individual voters' *preference* rankings over parties. Such preference information is not generally discernible from observed electoral returns (where voters typically only reveal a top choice and not their preference over pairs of lower ranked parties) and we therefore rely on self-reported preference rankings from surveys. Be-

[1]By their nature, the voter preferences over parties that enter in our analysis can be influenced by the universe of party features (policy relevant or not) perceived or observed by the voter, for example, the party leadership's private life, the party's organization and decision-making structure, the party's marketing and campaign strategies, etc.

Figure 1: Condorcet Winner in a Four-party System



Parties A, B, C, and D and the electorate (voters 1-5) are located on a two-dimensional space. If voters have Euclidean preferences then party A is a Condorcet winner: A majority (voters 1, 2, and 5) strictly prefer party A over B; a majority (voters 1, 2, 3, and 5) strictly prefer party A over C; and, a majority (voters 1, 2, and 3) strictly prefer party A over D.
Because Plott's (Plott (1967) symmetry conditions are not satisfied, party A is not even a core point in the entire two-dimensional space. E.g., it is majority defeated by a hypothetical party located at point E.

cause we have to confront the sampling uncertainty inherent in survey data,[2] we devise hypothesis tests of the null hypothesis that a party is a core party and similar tests of the null hypothesis that the party is a Condorcet party. These null hypotheses amount to a finite set of moment inequalities jointly holding, where the moments are fractions of the electorate each representing voters that weakly prefer the candidate core party over another party (or that strictly prefer the candidate Condorcet party over another party, respectively). To execute these tests we rely on the practical two-step procedure of Romano, Shaikh and Wolf

[2]As we discuss in section 3 where we present our test results, in a large number of cases parties fall comfortably within the category that either fails the criterion for a core party or satisfies it given broad polling margins of error. Our test procedure allows a rigorous way to adjudicate the evidence in the important and substantial minority of cases where mere ocular inspection of the survey results cannot be immediately convincing.

(2014), which makes both the computation of a test outcome and of a p-value practically feasible. Ours is one of many recent applications of moment inequalities in economics and political science, either for estimation and/or testing that includes, among others, Henry and Mourifie (2013), Kawai and Watanabe (2013), Park (2017), Iaryczower, Shi and Shum (2018), Kalandrakis (forthcoming).[3]

We implement these tests using CSES survey data from 56 countries, 196 surveys, and 193 national elections and perform a total of 1,176 tests for the parties included in each election/survey. For about half of these parties, we cannot reject the null hypothesis that the party is a core party. We cannot reject the hypothesis of a Condorcet party in 7.06% of these tests. Of the 196 surveys/elections under consideration, there always exists at least one party for which we cannot reject the core party null hypothesis. On the other hand, we cannot reject the Condorcet winner null for at least one party in 83 of these 196 surveys. While the test outcome yields a categorical classification of parties, the p-values associated with these tests provide finer resolution on likely core and Condorcet parties within the subset for which we fail to reject that status.

To flesh out the empirical content of these measures, we then go on to present two sets of correlational analyses. For reasons we elaborate later in the paper, we focus these analyses on the subset of our data drawn from parliamentary systems of government. In a first set of regressions, our test results are the dependent variable and right-hand-side variables include observable party attributes that might distinguish these parties and may be associated with a party's core or Condorcet winner status. We find, as expected, that the parties' electoral

[3]This is bound to be a partial list and we apologize for any omissions. A comprehensive review of the vibrant literature on moment inequalities is outside the scope of this study and we point the interested reader to Canay and Shaikh (2017) and Molinari (2020) for reasonably complete reviews of recent theoretical developments as well as for applications outside political economy.

performance such as their vote share and whether they won a plurality of the vote are strongly correlated with our measures of core and Condorcet parties. When it comes to measures of the ideological location of the parties in prominent one-dimensional left-right scales, we find that distance from the left-right median is (weakly) correlated with our measures of core and Condorcet parties, but there is no discontinuous location effect for a party at the median. Other variables have modest if any effects. While consistent with theoretical expectations, these results point to the conclusion that our tests isolate distinct party attributes over additional prominent observable party characteristics. They also suggest that, while the classic left-right socio-economic dimension accounts for an important component of party attribute variation, partisan spatial differentiation is often and/or likely multi-dimensional.

Assuming plausibility of these measures of core and Condorcet parties, a natural question is: Are such parties more likely to govern? There are strong normative reasons why we might want core and (certainly) Condorcet parties to be in the government but many actual voting aggregation methods do not guarantee it. Returning to our example from Figure 1, a proportional representation election with sincere voting would only yield 40% of the seats for the Condorcet party A (and the votes of 1 and 2), leaving open the possibility of a government coalition by parties B, C, and D which would jointly control 60% of the seats (receiving the vote of 3, 4, and 5, respectively). Thus, in a second set of regressions we use our test results as independent variables and our dependent variables are the party's participation in post-election government and the party's control of the prime minister position. Besides providing empirical evaluations of the normative expectation that core or Condorcet parties *ought to* govern, these results can also be thought of as evidence supporting the potential use of our empirical measurement procedure to *predict* government participation. In the notoriously fluid multiparty parliamentary system context these correlations suggest that an analyst could get a good leverage on identifying likely governing parties by running a survey ahead of the elections and executing our core and Condorcet tests.

Indeed, we find that core (and Condorcet) parties are significantly more likely to participate in the government, with an estimated boost in probability above 20%, even when a host of other control variables are included in the model. We also find that the chance that Condorcet parties participate in the government is higher but *not* statistically significantly different than that of core parties. When it comes to the premiership position, we estimate that (even accounting for other observables and election-specific unobserved heterogeneity) core parties are over 12% more likely to head the cabinet with this probability rising to over 40% when these parties are also Condorcet parties, the latter difference now being also statistically significant. Overall, our analysis bodes well for those who espouse the normative expectation that core and Condorcet parties should govern. It also suggests an important gradation of the empirical use of these measures: The more inclusive core party measure is most relevant when it comes to government participation, while the more discriminating Condorcet party measure adds to the predictive success of core party measures primarily when it comes to the control of the premiership position in the cabinet.

We emphasize that these findings, as those of most of this literature, do not have causal status. We do not view the absence of such 'causal' interpretation of our government participation findings as an indictment of these findings. As we discuss shortly, there is a long theoretical tradition that motivates our interest in the core. Our contribution is to introduce a new measurement method, to probe its face-validity, and to show that it can be useful at least in evaluating normative expectations about representation as well as for the purposes of empirical prediction (an exercise which is practically and conceptually different than establishing causation). Furthermore, while we do not preclude the possibility of causal analysis of core or Condorcet status on government participation, we caution that it is not obvious in this setting what is the empirical content of an 'ideal conditions' type of counterfactual that would otherwise grant causal status to such estimated effects? While there may be many omitted confounders that directly influence the probability that core parties participate in government, it is hard to imagine that such confounding party attributes are not inextricably related to voter preferences over parties. An idealized random or quasi-random assignment of core or Condorcet party status would require changing the majority fraction of voters that prefer one party over a competing party without changing the party attributes that determine these preferences or, alternatively, the existence of party attributes that influence voter preferences over parties (preferences almost definitionally over governing) but do not otherwise influence those parties' chance of government participation. A rehearsal of idealized causal experiments in simple settings is pretty revealing of the difficulties. For example, assume simple plurality rule and only two parties. An experiment-like switch of Condorcet status between the two parties amounts to switching the majority preference from one to the other party *without* changing party attribute confounders that influence governing chances of neither party.[4]

The need for empirically identifying core parties (according to some definition) has already been independently emphasized in the empirical literature on government formation, for example, in the influential study of determinants of government coalitions by Martin and Stevenson (2001) (page 35, footnote 2). The results we summarize above provide empirical support for the potential value of our notion of a core party and for our test procedures. On the theoretical side of the argument, there are a number of reasons why scholars of representative democracies might have an interest in core or Condorcet parties, and we discuss four of these reasons in what follows.

A first rationale for our interest in core parties is the classic cooperative game-theoretic justification for the core as a solution concept in strategic environments. This idea dates back to the origins of economic analysis as well as social choice theory. It was formalized in game theory by Gillies (1959), and the core as a solution concept is so prevalent thereafter

[4]Also note that the statement that this Condorcet status shift 'causes' government participation verges on the obtuse even if trivially true in this two-party, simple plurality setting.

that it is impossible for us to provide a detailed account of its use in the literature. Telser (1994) provides a relatively recent review in economics and we point the reader to Austen-Smith and Banks (1999) and the references therein for a social choice perspective. Within political science, a prominent account of government formation emphasizing the core can be found in Laver and Schofield (1990), among others. The broad appeal of the core as a solution set is that it provides a process- or institution-free prediction for outcomes *if* the choice set coincides with the set of alternatives that define the core, in our case the set of parties or their platforms. For that same reason, we do not privilege this justification (nor do we rule it out) because we believe that the set of possible government formation outcomes is broader than the finite set of points that the parties represent.[5] As we already discussed, a core party may (in fact, is likely to) be defeated by another alternative at which no party is located and for that reason we cannot be guaranteed any one party's position as the solution in a post-election policy-making game[6] once we allow political parties to engage in the quintessential political activity of compromising on policies other than the ones they advocate or profess.

A second reason to focus on core and Condorcet parties is because they may be expected to be more electorally successful, all else equal. By virtue of being weakly majority preferred in pairwise comparisons, core and (especially) Condorcet parties may often be the first vote choice of sincere voters or a second-best vote choice of strategic voters. But, we should also note in that connection that in section 4 we find that our measures of core or Condorcet party status have a statistically significant effect on government participation even after controlling for the party's seat share and its plurality status in the legislature.

[5]For example, see Thies (2001), Timmermans (2006), Strom, Muller and Bergman (2008), Martin and Vanberg (2011), and Bowler et al. (2016) for some of the ways coalition partners can sustain policy compromises away from their respective ideal points.

[6]Assuming a parliament where no single party controls a majority of seats.

A third reason to heed to the properties of core or Condorcet parties arises if we interpret them as sophisticated indicators of party popularity (over and above the mere vote or seat share these parties may garner). If the popularity of coalition partners factors into the calculus of the participants in the coalition, then the fact that core parties are undefeated in binary comparisons makes them attractive coalition partners, all else equal. Political parties' image is influenced by the coalitions in which they participate and parties would likely give a premium to partnering with a core party over a non-core party: By definition, a majority in the electorate strictly prefers another party over the potential coalition partner that is not a core party, while a core party carries no such liability.

Lastly, a fourth rationale is the relative position that core parties likely have in a spatial attribute space. Parties that avoid majority defeat by all other parties tend to, all else equal, be located towards the center of such a space, especially if that space is predominantly structured over a small number of prominent policy dimensions. Parties with such privileged location are attractive coalition partners to most other political parties due to affinity in the space (independent of popularity considerations), and are therefore likely to have an influential role in the policy compromises struck in the elected legislature. Indeed, a number of theories of government formation emphasize the locational advantage of such centrally located parties (e.g., Austen-Smith and Banks (1988); Baron (1991); Laver and Shepsle (1996)).[7] While this along with the last two reasons for focusing on core and Condorcet parties are premised on probable rather than universal attributes that co-vary with these parties, they provide in our view adequate justification for our interest in these party attributes and for our empirical findings.

We conclude this section with a review of additional related literature. The study

[7]As with our second rationale, we note that we estimate a significant effect for core parties on government participation even after we control for prominent measures of the parties' location.

of McDonald, Budge and Best (2012) is an important precursor to our work as they also use survey data to identify Condorcet parties. In addition to important details on how they determine majority preference, there are at least three main differences between their work and ours. First, we focus on core, in addition to Condorcet parties, and the two concepts are both definitionally and empirically (according to our findings) distinct. Second, we use reported *preferences* over parties, as required by the definition of the core and Condorcet winner concepts, whereas they impute such preference from responses on the probability of voting for a party. A voter may prefer party A over B and at the same time assign a higher probability to voting for B than A (in an election with more than two competing parties). Third, our study properly accounts for sampling uncertainty over social preferences. In part due to these differences, our findings regarding Condorcet parties differ from theirs in more than half of the countries they study.[8]

The work most close to ours, both methodologically and substantively when it comes to the task of identifying core parties is that by Kalandrakis (forthcoming). He develops moment inequality tests for the hypothesis that the set of parties can be lined up in one dimension so that the systematic preferences of the electorate satisfy a weak form of single-peakedness (weaker than that in Arrow (1951); Sen (1966)). While the main purpose of those tests is to recover (or otherwise reject the existence of) a one-dimensional scaling of the parties, they can also be interpreted as tests of a weak sufficient condition for the existence of a core party due to Dummett and Farquharson (1961). To the degree that they relate to our objectives in this paper, the tests in Kalandrakis (forthcoming) only constitute tests of a *sufficient* condition, whereas our core party procedure tests *necessary and sufficient* conditions. A second difference is that, because he allows for arbitrary error in survey responses, the outcome of the tests in Kalandrakis (forthcoming) does not immediately pin

[8]The comparison is not exact because their survey data is from 2004 which we compare to the nearest election survey we have available.

down the identity of any core parties, whereas our tests are specifically developed to identify such parties and directly deliver a measure for each party for which we conduct a test. While our analysis does not admit error in survey responses in the broad fashion that Kalandrakis (forthcoming) does, it does provide some safeguards for certain modal forms of response error as we further discuss in section 2.

A number of scholars have also proposed alternative subsets of political parties that for different reasons may play a significant role in post-election policy-making. Laver and Shepsle (1996) derive the concept of strong and very strong parties from their portfolio-centered theory of government formation. By definition, strong (and the subset of very strong) parties are different from our core parties. First, our concept is based on electorate-based social preferences versus party-based social preferences.[9] Our third rationale for studying core parties discussed above is premised on the idea that the electorate's preferences shape parties' coalition-making decisions. Second, in a parliament with J parties and D cabinet portfolio jurisdictions, our definition of a core party involves $J - 1$ social preference comparisons, whereas $J^D - 1$ such comparisons are required for strong parties. For example, with $J = 6$ parties and $D = 3$ dimensions, a core party is undefeated (in the electorate) against 5 other points whereas a strong party must be undefeated (in the legislature using party preferences) against 215 other points.

These differences leave open the possibility that our core parties and strong and very

[9] These party-based social preferences may differ from electorate-based social preferences on at least two grounds. First, vote and seat shares need not coincide. Second, even if vote and seat shares coincide, parties and their voters need not agree on how they rank other parties. It is certainly possible (in fact, likely) that individuals that vote for party A disagree on how they rank parties B and C. Furthermore, parties and voters may care about party attributes differentially so that, for example, party A may prefer party B over C, while most voters of party A may prefer party C over B.

strong parties may have independent effects on government participation. Unfortunately, we are unable to empirically evaluate such independent effects because, as far as we are aware, the main instance of empirical measure of strong parties in the literature (in Warwick (1996) and Martin and Stevenson (2001)) pre-dates the surveys we use in our analysis. With a similar objective of identifying parties that may play a significant role in post-election policy-making (and partly motivated by the practical difficulties of measuring strong and very strong parties), Powell (2019) proposes a set of *advantaged* parties (e.g., Powell (2019), pages 118-119). Accordingly, these parties' advantage may be due to size (fraction of seats and/or plurality status) and location (median status). Our measures of core parties have a significant effect on government participation even after the defining attributes of Powell's advantaged parties are controlled for in our analysis.

2 Setup

Consider J political parties, where $J \geq 2$. We use j and h to index parties ($j, h = 1, \ldots, J$) and reserve i to index individual voters, when necessary. Each individual voter i ranks the J parties so that, for each pair of parties j, h, voter i may either prefer party j over h, prefer h over j, or she may be indifferent between j and h. For every pair of parties j, h we can accordingly define a random variable for the preferences of a random voter in the electorate as

$$C(j,h) = \begin{cases} 1 & \text{if the voter prefers } j \text{ over } h \text{ or is indifferent between } j \text{ and } h, \\ -1 & \text{if the voter prefers } h \text{ over } j. \end{cases}$$

Recall that party j is a *core* party if there is no other party h that some majority strictly prefer over j. Then, a core party can be defined by the following $J-1$ moment inequalities

$$(C) \qquad \mathbb{E}[C(j,h)] \geq 0 \qquad \text{for all } h \neq j.$$

In this framework, if the expectation $\mathbb{E}[C(j,h)]$ is nonnegative, then the fraction (in the population) of voters that strictly prefer party h over party j is smaller than that of voters that weakly prefer party j over h.[10] It follows that party j is a core party if and only if inequalities (C) hold.

Related to the notion of a core party is that of a Condorcet winner, that is, a party j that is strictly majority preferred to every other party h. Every Condorcet winner is a core party, but there are core parties that are not Condorcet winners. We proceed similarly to formulate a test of whether party j is a Condorcet winner. We define for each party $h \neq j$ a random variable

$$W(j,h) = \begin{cases} 1 & \text{if the voter prefers } j \text{ over } h, \\ -1 & \text{if the voter is indifferent between } j \text{ and } h \text{ or prefers } h \text{ over } j. \end{cases}$$

[10] In effect, we assume that the finite electorate *is* the population and the survey samples we use are drawn as if with replacement from that population. We could alternatively assume that the electorate is drawn from an underlying population distribution conditionally (conditional on the configuration of parties) independently and we determine majority preference based on a moment of that distribution.

Now, if party j is a Condorcet party then[11]

$$(W) \qquad \mathbb{E}[W(j,h)] \geq 0 \qquad \text{for all } h \neq j,$$

that is, for all h, a (possibly weak) majority strictly prefer party j over party h. Just like the test for core parties, we test for Condorcet parties by testing the $J-1$ inequalities (W) as our null hypothesis against the alternative that $\mathbb{E}[W(j,h)] < 0$ for some $h \neq j$.

Inequalities (C) or (W) cannot be ascertained in the population from typical electoral returns because, strategic voting issues aside, these data typically only record top choices instead of individual preference rankings over all parties. We therefore resort to survey data. Suppose we have N voters independently drawn from the electorate and each voter i, $i = 1, \dots, N$, reports her preference ranking over the J parties. We extract these data from the so called party *sympathy* or party *thermometer* questions included in many national election studies' questionnaires[12] (and detail the exact data we use in our application of this test in the following section). To account for sampling uncertainty about the population moments $\mathbb{E}[C(j,h)]$, $\mathbb{E}[W(j,h)]$ inherent in such data, we formulate hypothesis tests. For core parties, the null hypothesis is that inequalities (C) hold against the alternative that $\mathbb{E}[C(j,h)] < 0$ for some $h \neq j$. For Condorcet parties, the null hypothesis is that inequalities (W) hold against the alternative that $\mathbb{E}[W(j,h)] < 0$ for some $h \neq j$. There are now a number of recently developed procedures to perform such tests and in our implementation, we use the *practical two-step procedure* of Romano, Shaikh and Wolf (2014).[13] These authors

[11] Sufficiency would follow if we require inequalities (W) to hold strictly. There is no real practical consequence to the weak inequality formulation, though, and our testing framework is designed to handle weak inequalities.

[12] While we only extract ordinal information from these survey responses, they may in fact reflect cardinal preferences (e.g., Eggers and Vivyan (2020)).

[13] In our setup with a small number of $J-1$ moment inequalities in each test, an alternative

establish a number of valid choices for the test statistic used to implement this test, and we use results from the MAX statistic implementation in the main text.[14]

Based on these tests, we can construct both categorical as well as continuous measures of core and Condorcet parties. For a categorical measure, we use the rejection decision of the test procedure at the conventional significance level of $\alpha = 0.05$. We call a party for which we fail to reject the null hypothesis (C) or (W) a core or Condorcet party, respectively. We use the p-value of the hypothesis test as a continuous measure. The two-step procedure of Romano, Shaikh and Wolf (2014) allows computation of a p-value, which in our setup is defined as the smallest level of significance α at which the test is rejected (see the supplemental appendix of Romano, Shaikh and Wolf (2014), page 6). Exploiting a monotonicity in the test, we compute these p-values using bisection at mild additional cost and at a precision past the second decimal point.

The categorical measure is conceptually simple and provides a stark binary summary. On the other hand, a larger p-value, by construction, indicates the observed evidence is more consistent with the party being a core or Condorcet party, and the parties we deem as *not* being core or Condorcet using our categorical measure have a p-value less than 0.05. Because most of the variation in the computed p-values is within the group of parties the categorical

feasible test with similar properties is the recommended procedure of Andrews and Barwick (2012). We use the test of Romano, Shaikh and Wolf (2014) because of the added advantage that it allows us to easily compute a p-value. This literature is fast-evolving, and the new tests developed by Gregory Cox and Xiaoxia Shi (Cox and Shi (forthcoming)) also deliver comparable properties at reduced computational cost.

[14]We also carried these tests with alternative statistics but we did not encounter noticeable differences in test outcomes (see Appendix A) or in our regression analyses. Kalandrakis (forthcoming) also finds little difference between the various implementations, and favors the MAX statistic for its numerical stability and cost of computation.

measure deems core or Condorcet, this continuous measure yields finer information primarily *within* the group of core and Condorcet parties. Not surprisingly, our regression analyses later in this paper confirm that the more refined information in the computed p-values does improve model fit.

2.1 Interpretation with respondent error

The tests we just presented naturally come with statistical error due to finite sample implementation of the procedure.[15] Of course, the properties of these tests also hinge on the quality of the data we use, notably the assumption that the preference data we obtain in our sample are themselves reported without error. If we entertain the possibility that survey responses contain error, then our present testing framework can accommodate a specific – and likely modal – form of such response error. If respondent i prefers party j over h (or h over j), she may report (in error) that she is indifferent between j and h. If that is the case and no reported *strict* preference is in error, then our test for core parties is a *conservative* test: We may fail to reject the null that party j is a core party because some respondents that strictly prefer party h over j may have reported they are indifferent. Conversely, our test for a Condorcet winner is a *liberal* test of both the Condorcet winner hypothesis and of the hypothesis that the party is a core party, because respondents that strictly prefer party j over h may have reported they are indifferent instead (and we will then tend to reject the null of a Condorcet party). If response error is limited to that form, then results from the two tests give us an upper and lower envelope on the set of core parties: The actual set of core parties will tend to be a subset of the set of parties for which we fail to reject the core party test, and a superset of the set of parties for which we fail to reject the Condorcet

[15]See Romano, Shaikh and Wolf (2014), notably Theorem 1, their Remark 2, page 1984, and the reported Monte Carlo power calculations.

winner test.[16]

While we cannot rule out more egregious forms of response error at the outset, it is generally expected that many of the respondents in those surveys tend to report indifference when committing error, for example, by correctly reporting their most preferred party (or parties) and then placing many of the parties they rank lower at the same indifference set.[17] We should add that this type of response profile is only *presumed* to be in error: Without additional evidence, it is impossible to ascertain whether individual respondents are committing error or whether they actually reveal (practical) indifference towards some parties or even politics altogether.

The survey literature also broadly identifies *social desirability bias* as another significant source of error in responses. In our case, two forms of such error seem particularly relevant: Respondents tend to under-report their preference for parties that are marginalized, unconventional, or otherwise outside the mainstream; or they may tend to over-report their preference for successful parties. Both of these forms of bias are of concern especially when they take the form of strict preference error in responses. We should point out that the first of these two forms of error is likely less consequential for our results because a party that is socially undesirable is almost by definition not a party that a majority weakly prefer over all other parties (a core party), and therefore the fact that the number of respondents that report preferring such parties is smaller than the actual number is unlikely to lead to the wrong test decision.

[16]Put another way, if we are testing party j and assume all respondents who are indifferent between j and any h actually prefer party j, then our core test is also a Condorcet test. If, on the other hand, all respondents who are indifferent between j and any h actually prefer h, then our Condorcet test is also a core test.

[17]But see also Ansolabehere and Brady (1989) who offer another take on self-reported voter indifference.

3 Data and Test Results

We implement these test procedures on survey data from all five modules of The Comparative Study of Electoral Systems (2019, 2020) (CSES). These data comprise 196 surveys timed around national elections across 56 countries with the earliest election taking place in 1996, and the latest election in 2019. We use the thermometer/sympathy questions from these CSES surveys (with one such separate question per party included in the survey) which ask respondents how much they like each political party on an integer scale (typically 0-10). Higher values indicate that the respondent likes that party more and we use these responses in a straightforward manner to compute sample realizations $c_i(j, h), w_i(j, h)$ of the random variables $C(j, h), W(j, h)$ we introduced in section 2. By design, these surveys include such thermometer/sympathy questions for most parties winning substantial vote shares. Crucially for our analysis of government participation in parliamentary systems, the overwhelming majority of these surveys include virtually all the parties that earn seats in the legislature (see Table C.3 of Appendix C), which arguably is the relevant set of parties on which to apply our definition of the core when it comes to government participation outcomes.[18]

For each test we run, we report both a binary test outcome indicating whether or not we failed to reject the core or Condorcet null at the significance level of $\alpha = 0.05$, as well as the corresponding test p-value as a continuous measure. The surveys often (but not always) provide different weighting schemes for survey responses and, wherever possible, we use sample weights to carry out weighted versions of these tests. Because there appears to be wide variability in the choice to report, the types of weights, and the documentation of

[18]Even if extra-parliamentary parties are somehow relevant in the post-election government formation process, they are almost surely very small parties that are unlikely to rise to core party status or to affect the core party status of other parties.

these weights included in these data, we focus our analysis in the main body of the paper on the unweighted tests. The additional weighted tests yield similar results.

Our data comprise of 1,245 unique party-survey pairs. We exclude parties for which survey responses for sympathy scores are less than 75% of the survey responses for the party that received the most responses in the survey. This effectively excludes very small parties – most often extra-parliamentary – or parties that are predominantly present in a specific region of the country (such as the Croatian Civic Initiative in Montenegro or the Galician Nationalist Bloc in Spain).[19] These exclusions leave us with 1,176 party-survey pairs. Table 1 provides a summary of the test results carried out for these parties.[20]

Table 1: Summary of test results

Total parties tested	Core parties	%Core parties	Condorcet parties	% Condorcet parties	Total surveys	Surveys with core party	Surveys with Condorcet party
$1,176$	566	48.13	83	7.06	196	196	83

Notes. Number of parties for which we fail to reject the core or Condorcet null hypothesis. All tests carried out using the two-step procedure of Romano, Shaikh and Wolf (2014), with the MAX statistic, $\alpha = 0.05, \beta = 0.005, B = 20,000$.

From Table 1 we see that we cannot reject the core party null at the significance level of 0.05 for 566 out of $1,176$ or 48.13% of parties in the sample. In fact, in each of the 196 surveys there exists at least one party for which we fail to reject the core party null hypothesis. On the other hand, we cannot reject the Condorcet winner null hypothesis for only 83 out of $1,176$ parties or 7.06% of the parties in the sample. These parties are always a subset of the set of parties for which we fail to reject the core null hypothesis (similarly,

[19]The 75% cutoff eliminates 69 of the 1,245 parties. An 80% threshold would have eliminated 109 parties, whereas a 70% threshold 47 parties.

[20]We provide a summary of these test results disaggregated by country in Appendix B.

the core test p-values are always weakly larger than the Condorcet test p-values for the same party). Furthermore, whenever we fail to reject the Condorcet null hypothesis for a party in a survey, we reject that hypothesis for all other parties in that survey. Therefore, the 83 parties for which we cannot reject the Condorcet winner null correspond to 83 distinct surveys, leaving 113 surveys for which the Condorcet party null is rejected for all parties in the survey. Though the number of parties varies by election and country, the modal number of parties per survey is $J = 6$. According to the above aggregate results, of the $J = 6$ parties per survey, almost 3 pass[21] the binary core test, and less than 1 (on average 0.42) pass the stricter Condorcet winner test.

Table 2: Distribution of core and Condorcet test p-values

	[0,0.01]	(0.01,0.05]	(0.05,0.25]	(0.25,0.5]	(0.5,0.75]	(0.75,0.95]	(0.95,1]
Core	581	29	46	22	31	29	438
Condorcet	$1,077$	16	9	12	5	7	50

Notes. p-values in the universe of $1,176$ core and Condorcet tests performed. Parties for which the null hypothesis is rejected are in the first two columns. All p-values computed using the two-step procedure of Romano, Shaikh and Wolf (2014) with the MAX statistic, $\beta = \alpha/10, B = 20,000$.

In Table 2, we provide further information on the distribution of the core and Condorcet test p-values. The first two columns correspond to parties with p-value less than 0.05, i.e., those that do not pass the core or Condorcet test. Within that set, it becomes evident that in most cases we would also reject the core or Condorcet null hypothesis at the $\alpha = 0.01$ level of significance (for 581 out of 610 parties that fail the core test and for $1,077$ out of $1,093$ parties that fail the Condorcet test). A similar skew is evident within the group of parties that pass the core and Condorcet tests: 438 out of the 566 parties that

[21] From here on, we abuse terminology and state that a party *passes* a test instead of stating that we *fail to reject the null hypothesis* for that party. A more detailed breakdown of these results by country appears in Appendix B.

pass the core test feature a p-value larger than 0.95, whereas the corresponding number for Condorcet parties is 50 out of 83. These are cases where there is minimal evidence against the null hypothesis. This bimodal distribution is quite intuitive: Given polling margins of error, most parties either clearly violate one of the majority comparisons (inequalities) or comfortably satisfy all the inequalities within each test. The empirically difficult cases are those that are not safely (and jointly) within the polling margin of error and it is within this minority of cases that our test procedure provides the most valuable information. This concerns, in particular, the 128 parties with a core p-value between 0.05 and 0.95, or 22.61% of the 566 parties that pass the core test; and 33 parties with a Condorcet p-value between 0.05 and 0.95, or 39.76% of the 83 parties that pass the Condorcet test. Using these p-values instead of the coarser test outcome as our empirical measure, we obtain substantially refined information within the respective groups of core and Condorcet parties.

4 Test results & other observables

In this section, we evaluate the empirical content and potential use of these test results by studying how they correlate with a number of key observables. First, as a face-validity test we study how our measures relate with party-specific attributes that are thought to be associated with core or Condorcet party status. Second, we evaluate empirically the normative (and theoretical) expectation that core or Condorcet party status correlates with inclusion in the post-election government and with assuming the role of the head of government. Recall that being a core or Condorcet party is (definitionally) an attribute determined by the alignment of preferences in the electorate. As such, the goal of the regressions we report is *not* to isolate variables that 'cause' either this alignment of preferences (to the degree that such an exercise even makes sense) or to 'cause' participation in government. Uncovering such causal status is likely challenging in the best circumstances, as we discuss in page **??**. Rather, we view these analyses as evidence to probe the validity scope of our test procedures and results.

We find it necessary to restrict the sample for the purposes of these analyses. The main relevant restriction is that we focus on data from non-presidential systems. In addition to the fact that presidential systems feature possibly distinct patterns of party alignment and (crucially) government formation, many of the CSES surveys in presidential systems are timed around a presidential instead of a lower house election, thus preventing comparable measurement of the parties' electoral performance. This along with additional restrictions we detail in Appendix C, leave us with 121 out of 196 surveys and a maximum of 761 out of 1,176 party/survey observations.

We start with some summary statistics on key variables in Table 3, using the binary test outcomes for clarity of exposition. Of the 761 parties across surveys in the sample, 335 pass the core test. Of these 335 parties, about 279 only pass the core test, while 56 also pass the Condorcet test. Reading Table 3 from left to right, core parties, core-only parties, and Condorcet parties are more likely to have been in the outgoing cabinet government (incumbent column), to be the median along a left-right scale in the legislature[22] (legislative median column), to garner a plurality of the seats in the legislature (seat plurality column), to garner a majority of seats in the legislature (seat majority column), to participate in the post-election cabinet (in gov. column), and to control the premier office in the post-election cabinet (PM party column). While the difference between core-only parties and other non-core and non-Condorcet parties is less pronounced for the median and seat majority variables, the overall pattern clearly suggests that core and Condorcet parties are a special subset of the set of political parties in these surveys. This difference is most stark in the government participation variable, where well over half of the core-only parties and 92.9% of Condorcet parties end up in the cabinet (while the baseline fraction is only 35.7% for all parties); and

[22]We detail our construction of the median variable in Appendix C.

Table 3: Summary statistics for all, core, and Condorcet parties

	Total	Incumb. party	Legislative median[a]	Seat plurality[b]	Seat majority	In gov.	PM party[c]
All	761	237	119.6	122	17	272	113
%	100	31.1	15.7	16.0	2.2	35.7	14.8
Core parties	335	142	88.2	118	17	198	112
%	100	42.4	26.3	35.2	5.1	59.1	33.4
Core parties only	279	112	55.8	70	8	146	67
%	100	40.1	20.0	25.1	2.9	52.3	24.0
Condorcet parties	56	30	32.5	48	9	52	45
%	100	53.6	58.0	85.7	16.1	92.9	80.4

Notes. Key summary statistics for the subsample of non-presidential democracies considered in section 4 (see Appendix C). Core-only parties are parties that pass the core test, but do not pass the Condorcet test. Within the four categories of all, core, core-only, and Condorcet parties, the first row gives the count per column attribute and the second row the percent this count represents as a fraction of the total number of parties in this category as reported in the first column.

[a] Our median measure takes fractional values when more than one parties may logically be at the median position; some of these median parties are not in the sample (Spain 2004, 2008, Italy 2006, and Slovenia 1996).

[b] Iceland 2013 has two parties tied with a plurality of seats.

[c] The Prime Ministers in Hungary 2002, Italy 2006, Italy 2016, Romania 1996 were independents, and Switzerland has no equivalent position.

(for Condorcet parties) the Prime-minister variable where 80.4% of these parties end up holding that office.

Test results and party observables

In this subsection we report on our analysis relating our core and Condorcet test measures with theoretically related party attributes. Throughout we focus on results for our continuous p-values measure rather than the categorical test result measure.[23] Following our

[23]Though often less pronounced, our results are similar if we use the binary test outcomes rather than the p-values (available upon request).

discussion in the introduction, we expect at least two categories of observables to be useful in this exercise: Those that indicate that a party is more popular, and those that indicate that the party is centrally located, *ceteris paribus*.

In the first category of relative popularity measures we use electoral returns from the lower house election (LH vote share). We incorporate lower house vote share data directly from the CSES dataset.[24] We use the vote share of the party as a continuous measure and we also add a dummy variable (Electoral Plurality) for the party that won a plurality of the vote in these elections. The expectation is that both variables correlate positively with both the core and Condorcet party p-values. Turning to measures of relative location of the parties, the CSES includes a question asking respondents to place each party on a 0-10 left-right scale in most surveys. We use the average of these voter scores as the left-right location of the parties to compute the electoral median party in one dimension as well as the distance from the median for each party.[25] Because the parties for which the left-right scores are available need not receive 100% of the votes, we determine possible medians in a conservative way and we detail those calculations in Appendix C, where we also detail how our results are robust to more direct alternative calculations of the median.

We should emphasize that we are agnostic about the dimensionality of the party space in our test setup. In the special case when parties can be placed on a single dimension and voters have weakly single-peaked preferences on this dimension, then at least one of the parties (certainly the party at the median along that dimension) is a core party. If,

[24]There are 14 fewer observations in the regressions of Table 4 compared to Table 3 because several parties in certain surveys contested the elections together as a coalition and their disaggregated LH vote share is not available.

[25]We also conduct these analyses using the Comparative Manifesto Project (CMP) (Volkens et al. 2019) *RILE* (right-left) party scores instead of the CSES voter averaged left-right scores (available upon request).

in addition, the preferences are single-peaked, then a unique median party is a Condorcet winner. These theoretical results do not automatically guarantee that the median in any one-dimensional scale is in fact a core or Condorcet party because other dimensions (e.g., foreign policy, environment, ethnicity, etc.) may independently capture variation in preferences. But even if the ambient space of party attributes is higher-dimensional, we expect that a party closer to the median in a prominent left-right dimension may correlate with core and Condorcet winner status.

The last variable we include in this analysis is the incumbency status of these parties. Specifically, we construct a dummy government incumbency variable indicating whether a party participated in the cabinet before the election or not. We obtain these data from CSES (after module 2) and supplement them using data from the ParlGov database (Döring and Manow 2019) and from the Party Government Data Set (Woldendorp, Keman and Budge 2000). We estimate all models using OLS and cluster standard errors at the survey level.

Table 4 presents the results. Models (1) and (2) do not account for any unobserved heterogeneity and the core party measure is the dependent variable for model (1), while the Condorcet winner measure is the dependent variable in model (2). Models (3) and (4) are the corresponding models accounting for party-specific unobserved heterogeneity by including party fixed effects, while models (5) and (6) also include survey/election fixed effects. Focusing on models (1) and (2), we remark first that the independent variables in these models account for about one third of the variation in our core and Condorcet measures according to the R^2 coefficient: Our core and Condorcet party measures are informative and not mere reformulations of known or otherwise observable measures of the parties' electoral success or party location variables.

Turning to the specific independent variables in these regressions, across all models, we see that the lower house vote share correlates positively with both the core and Condorcet party measures. Depending on the model, a ten percentage point increase in vote share

Table 4: Test results and other observables

	Dependent variable:					
	Core p-value	Condorcet p-value	Core p-value	Condorcet p-value	Core p-value	Condorcet p-value
	(1)	(2)	(3)	(4)	(5)	(6)
LH vote	0.019***	0.005***	0.022***	0.010***	0.025***	0.011***
	(0.002)	(0.001)	(0.004)	(0.003)	(0.003)	(0.002)
Electoral Plurality	0.165***	0.198***	0.163**	0.155***	0.134**	0.151***
	(0.059)	(0.043)	(0.074)	(0.048)	(0.053)	(0.038)
Electoral Median	0.098*	0.146***	0.060	0.054	0.060	0.069
	(0.054)	(0.043)	(0.077)	(0.067)	(0.054)	(0.050)
Distance from median	−0.035**	−0.001	−0.047*	−0.017	−0.055***	−0.015
	(0.014)	(0.006)	(0.028)	(0.014)	(0.019)	(0.013)
Incumb. pty.	−0.003	−0.038**	−0.109**	−0.020	−0.101***	−0.029
	(0.036)	(0.019)	(0.045)	(0.028)	(0.034)	(0.024)
Constant	0.122***	−0.054***				
	(0.038)	(0.015)				
Observations	747	747	747	747	747	747
R^2	0.391	0.356	0.735	0.637	0.836	0.718

Notes: *p<0.1; **p<0.05; ***p<0.01. Sample restricted to non-presidential democracies as detailed in Appendix C. Models (1) and (2) are OLS, models (3) and (4) include party fixed effects, and models (5) and (6) include party and survey fixed effects. Standard errors are clustered by survey in all models.

translates to an increase of between 0.19 to 0.25 (across models (1), (3) and (5)) for the core party p-value and a 0.05 to 0.11 increase (across models (2), (4) and (6)) in the Condorcet p-value. Winning a plurality of votes is also strongly correlated with both the Condorcet and core measures: An increase between 0.134 to 0.165 in the core p-value and an increase of 0.151 to 0.198 in the Condorcet p-value.

Turning to the location variables, while the coefficients on the median variable are positive in all models, they are only statistically significant for the models without fixed effects. Distance from the median also has the expected negative sign, suggesting that parties further away from the median party are less likely to be core and Condorcet parties. The significance in the estimated effects exhibits somewhat different pattern between the core and Condorcet measures, as the effect is significant for the core measure in the simple OLS model (1) and in model (5) with both party and survey fixed effects, and only marginally significant in model (3) with just party fixed effects. It is never significant when it comes to Condorcet parties. Proximity to the median (according to models (1) and (5)) has a continuous effect on the core party measure without a significant discontinuous jump when reaching the median position. A similar interpretation is not supported for the Condorcet measure, where median status only (if at all) seems to have an effect.

The overall picture from these results is consistent with theoretical expectations when it comes to the sign of the estimated effect and (possibly) when it comes to the significance, especially because there is no guarantee that these one-dimensional scales fully account for heterogeneity of preferences over the parties in the sample. If we maintain that politics across all these elections are one-dimensional, another possible explanation for the lack of robustness in the estimated effects is that our sample exhibits small variation in these left-right location measures for each party across elections in the same country, which may account for the erosion of statistical significance once party fixed effects are accounted for in the model. We should also point out that the findings for these location variables are the least robust to the use of the alternative CMP $RILE$ scores (these results are available upon request) with which we find that being at the median is significantly correlated across all models with only the Condorcet party status, while the distance from the median is not significantly correlated with either core or Condorcet party measure.

Finally, with regard to the incumbency variable, participation in the outgoing cabinet has a negative effect, which is once more sparsely significant for either the core or for the Condorcet parties. These estimates may suggest that parties in government may pay a small popularity penalty (on average), though both the size of the sample and the fact that it incorporates a significant fraction of observations from the recent world financial crisis does not allow for any forceful interpretation of these coefficients.

Government participation and Premiership

With the above results encouraging as far as the face validity of our measures, we now use these measures to evaluate the (normative and theoretical) expectation that core and Condorcet parties are more likely to participate in the post-election government and/or control of the prime-minister office. Our dependent variables are binary taking the value one if the party in question was included in the post-election government, and zero otherwise for the party participation models and take the value one if the head of the cabinet is a member of the party and zero otherwise. We compile these data as we did for the pre-election incumbency variable combining information from the CSES, the ParlGov database (Döring and Manow 2019), and from the Party Government Data Set (Woldendorp, Keman and Budge 2000). Throughout this section, we further restrict the sample to the subset of surveys/elections that did not result in a single party controlling a majority of seats in parliament, leaving 104 (out of 121) surveys with 680 (instead of 761) party-election pairs. These are the interesting cases, as a party that wins a majority of seats is naturally expected to be in the government and control the premiership.[26]

Our main interest is whether a party with a higher core and/or Condorcet p-value is more likely to be included in the post-election cabinet and/or capture the premiership.[27] To these main variables of interest, we add variables analogous to those included in Table 4, but shift the composition of these measures to reflect legislative instead of electoral quantities (that is, we use seat shares instead of vote shares). We therefore control for each party's seat share in the lower house (LH seat share), along with dummy variables indicating whether the party controls a plurality (Seat plurality). These variables are broadly acknowledged as relevant in theories of coalition formation (e.g., Martin and Stevenson (2001)) either as having a direct effect or as having an indirect effect by boosting a party's chances of *formateur* selection (e.g., Diermeier and Merlo (2004), Fujiwara and Sanz (2020)).

We similarly expect measures of spatial centrality to have an effect as emphasized by both the theoretical and the empirical literature (Austen-Smith and Banks 1988; Baron 1991; Laver and Shepsle 1996; Martin and Stevenson 2001). The legislative median is calculated similarly to the electoral median along the voter averaged left-right scores, but now using seat instead of vote shares. The distance from the median now reflects the distance of the party's left-right score from the calculated legislative median. Finally, there exists at least some evidence in the literature that parties in the outgoing cabinet may have a higher chance of inclusion in the new cabinet, and we include a dummy indicating whether the party participated in the pre-election cabinet. For the models of the premiership position, we use

[26]This expectation is borne out in our sample. If these majority parliament observations are included in our analysis, a seat majority dummy variable exhibits complete quasi-separation (the dependent variable is one whenever that variable is equal to one) and, e.g., the logit ML estimator does not exist.

[27]The obvious additional caveat that these test outcome measures come with measurement error (as do a host of other variables in these regressions) is duly acknowledged.

a similar incumbency dummy variable, this time measuring whether the party controlled the prime-minister office in the outgoing government.

With both of the dependent variables in this section being categorical, we face a wealth of modeling choices. For models without fixed effects we report both the OLS linear probability models (LPM) and the corresponding logit. Because both party and survey groups in our sample are small (there are at most six surveys per party but much fewer typically, and roughly six parties per survey) we only report LPMs for models with fixed effects. These directly quantify the marginal effects of interest while also consistently accounting for unobserved heterogeneity. In all reported models, we cluster standard errors at the survey level.

We start by discussing results on our government participation models which are reported in Table 5 and 6. Table 5 includes four simple OLS and four logit models with identical specifications. Table 6 includes an additional eight models, four with party fixed effects and another four with both party and survey fixed effects. Within each set of four models, the first models (models (1) and (5)) only include our measures of core and Condorcet parties, while the remaining three models include all the other explanatory variables and one or both of our measures. Starting with the sparser models, we see that both core and Condorcet measures are statistically and (gauging from the OLS marginal effects) substantively significant: A party that has a core p-value of one and a Condorcet p-value of zero has over 40% chance of participation in the cabinet (model (1) of Tables 5 and 6, and model (5) of Table 6). On the other hand a party with a Condorcet p-value of one (which by necessity must have a core p-value of one), has over 75% (and as much as 95.7%) chance of being in the cabinet in the corresponding models. These estimated effects shrink once we control for other variables but the core measure estimate remains statistically significant across all models (including the logit models of Table 5). The corresponding probabilities from the most complete models (model (4) in Tables 5 and 6, and model (8) in Table 6) are around

Table 5: Government participation

	OLS				*Logistic*			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Core p-value	0.408***	0.237***		0.236***	1.815***	1.158***		1.133***
	(0.040)	(0.048)		(0.048)	(0.193)	(0.232)		(0.227)
Condorcet p-value	0.356***		0.038	0.029	2.527***		1.414	1.089
	(0.056)		(0.074)	(0.071)	(0.812)		(1.239)	(1.038)
LH seats		0.003*	0.007***	0.003		0.023*	0.042***	0.021*
		(0.002)	(0.002)	(0.002)		(0.012)	(0.011)	(0.012)
Seat Plurality		0.228***	0.260***	0.222***		1.280***	1.255***	1.163**
		(0.074)	(0.082)	(0.079)		(0.476)	(0.476)	(0.494)
Legislative Median		0.142**	0.141**	0.138**		0.613	0.606*	0.600
		(0.061)	(0.063)	(0.061)		(0.376)	(0.367)	(0.377)
Distance to median		−0.068***	−0.079***	−0.068***		−0.505***	−0.541***	−0.500***
		(0.013)	(0.013)	(0.013)		(0.108)	(0.105)	(0.108)
Incumbent party		0.096*	0.099*	0.096*		0.539*	0.537*	0.549*
		(0.051)	(0.052)	(0.051)		(0.294)	(0.290)	(0.295)
Constant	0.199***	0.267***	0.309***	0.268***	−1.380***	−1.004***	−0.803***	−0.991***
	(0.020)	(0.038)	(0.039)	(0.037)	(0.120)	(0.232)	(0.225)	(0.230)
Observations	680	680	680	680	680	680	680	680
R^2	0.221	0.327	0.295	0.327				
Log Likelihood					−371.005	−324.348	−334.537	−323.393

Notes: *p<0.1; **p<0.05; ***p<0.01. Sample restricted to non-presidential democracies and elections that produced a minority parliament, as detailed in Appendix C. Models (1)-(4) are OLS and models (5)-(8) are logistic. Standard errors are clustered by survey in all models.

Table 6: Government participation (including fixed effects)

	Party fixed effects				*Party and survey fixed effects*			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Core p-value	0.414***	0.177**		0.181**	0.485***	0.224***		0.214***
	(0.081)	(0.077)		(0.079)	(0.079)	(0.075)		(0.076)
Condorcet p-value	0.509***		0.100	0.118	0.472***		0.158	0.104
	(0.104)		(0.131)	(0.127)	(0.101)		(0.111)	(0.111)
LH seats		0.012***	0.015***	0.011***		0.013***	0.017***	0.012***
		(0.003)	(0.003)	(0.004)		(0.003)	(0.003)	(0.003)
Seat Plurality		0.335***	0.347***	0.315***		0.306***	0.311***	0.291***
		(0.094)	(0.102)	(0.100)		(0.073)	(0.081)	(0.078)
Legislative Median		0.063	0.041	0.054		0.052	0.028	0.045
		(0.102)	(0.105)	(0.102)		(0.076)	(0.080)	(0.076)
Distance to median		−0.106***	−0.116***	−0.104***		−0.112***	−0.123***	−0.111***
		(0.031)	(0.033)	(0.030)		(0.024)	(0.026)	(0.024)
Incumbent party		−0.142*	−0.158*	−0.142*		−0.121*	−0.139**	−0.121*
		(0.085)	(0.086)	(0.084)		(0.065)	(0.065)	(0.065)
Observations	680	680	680	680	680	680	680	680
R^2	0.621	0.689	0.683	0.690	0.665	0.727	0.721	0.727

Notes: *p<0.1; **p<0.05; ***p<0.01. Sample restricted to non-presidential democracies and elections that produced a minority parliament, as detailed in Appendix C. All models are OLS, models (1)-(4) include party fixed effects and models (5)-(8) include party and survey fixed effects. Standard errors are clustered by survey in all models.

20% probability of inclusion in the cabinet for core-only parties (when our Condorcet measure is zero); while Condorcet parties (with p-value equal to one) have an estimated boost of 2.9-11.8% in the probability of being in the cabinet.

These estimates are consistent with the evidence we presented in Table 3, though here we estimate a boost in the probabilities of inclusion in the cabinet even after we control for a host of other possible correlates of government participation (and for unobserved party/survey-specific heterogeneity). Furthermore, a higher Condorcet p-value does not provide a statistically discernible boost to the probability of being included in the post-election cabinet. This is so across all complete models (models (4) and (8)) reported in Tables 5 and 6. The non-significant coefficient for the Condorcet measure in these models implies that there is no statistically significant increase in the Condorcet party's government inclusion chance over and above what is already implied by these parties' core party status.[28]

The main other variables that are statistically significant in Tables 5 and 6 are seat plurality and distance from the median (consistently across models), with seat plurality resulting in a substantial boost to the probability of inclusion in the cabinet. Seat share (LH seats) and being at the median location also have a positive effect. Incumbency has a positive effect in models without fixed effects and a negative effect otherwise. Statistical significance is sparse or absent in all three of these variables.

Next, we turn to the corresponding models for the prime-minister office, which are reported in Tables 7 and 8. The model specifications in these tables mirror those for the government participation models. The findings for the core party measure are very similar when it comes to statistical significance: A core party is statistically significantly more likely to control the prime-minister office, though the magnitude of that effect is now smaller and closer to about 6-10% in the OLS models. On the other hand, the more discerning Condorcet party measure is now both statistically and substantively boosting a party's chance of holding

[28]Recall that a core test p-value is always weakly larger than a Condorcet p-value.

Table 7: Premiership

	OLS				Logistic			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Core p-value	0.267***	0.071***		0.069***	3.578***	1.826***		1.782***
	(0.021)	(0.023)		(0.022)	(0.435)	(0.590)		(0.544)
Condorcet p-value	0.626***		0.216**	0.214**	2.984***		0.891	0.767
	(0.064)		(0.087)	(0.087)	(0.534)		(0.892)	(0.817)
LH seats		0.004***	0.005***	0.004**		0.092***	0.107***	0.087***
		(0.001)	(0.001)	(0.001)		(0.016)	(0.016)	(0.018)
Seat Plurality		0.516***	0.481***	0.469***		2.041***	2.190***	1.961***
		(0.077)	(0.084)	(0.085)		(0.526)	(0.545)	(0.559)
Legislative Median		−0.016	−0.039	−0.040		−0.855	−1.024	−0.959
		(0.037)	(0.036)	(0.036)		(0.575)	(0.626)	(0.592)
Distance to median		−0.018***	−0.021***	−0.017***		−0.459**	−0.547***	−0.438**
		(0.006)	(0.006)	(0.006)		(0.200)	(0.191)	(0.205)
Incumbent premier		0.068	0.067	0.071		0.434	0.459	0.457
		(0.058)	(0.058)	(0.057)		(0.511)	(0.517)	(0.528)
Constant	0.005	−0.006	0.014	0.001	−4.511***	−5.193***	−4.247***	−5.077***
	(0.005)	(0.014)	(0.013)	(0.013)	(0.413)	(0.620)	(0.348)	(0.545)
Observations	680	680	680	680	680	680	680	680
R^2	0.374	0.585	0.592	0.597				
Log Likelihood					−166.721	−108.426	−113.321	−107.729

Notes: *p<0.1; **p<0.05; ***p<0.01. Sample restricted to non-presidential democracies and elections that produced a minority parliament, as detailed in Appendix C. Models (1)-(4) are OLS and models (5)-(8) are logistic. Standard errors are clustered by survey in all models.

Table 8: Premiership (including fixed effects)

	Party fixed effects				*Party and survey fixed effects*			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Core p-value	0.294***	0.110***		0.120***	0.317***	0.150***		0.115***
	(0.044)	(0.041)		(0.042)	(0.042)	(0.046)		(0.042)
Condorcet p-value	0.668***		0.321***	0.332***	0.694***		0.391***	0.361***
	(0.096)		(0.123)	(0.122)	(0.083)		(0.106)	(0.102)
LH seats		0.007**	0.007**	0.004		0.007***	0.008***	0.006**
		(0.003)	(0.003)	(0.003)		(0.003)	(0.002)	(0.002)
Seat Plurality		0.597***	0.564***	0.543***		0.581***	0.539***	0.528***
		(0.097)	(0.105)	(0.104)		(0.075)	(0.081)	(0.080)
Legislative Median		−0.017	−0.049	−0.041		−0.019	−0.052	−0.043
		(0.057)	(0.060)	(0.058)		(0.045)	(0.049)	(0.047)
Distance to median		−0.031*	−0.032*	−0.024		−0.031**	−0.033**	−0.026*
		(0.017)	(0.018)	(0.017)		(0.015)	(0.015)	(0.015)
Incumbent premier		−0.064	−0.082	−0.072		−0.068	−0.089	−0.078
		(0.083)	(0.079)	(0.079)		(0.064)	(0.060)	(0.061)
Observations	680	680	680	680	680	680	680	680
R^2	0.687	0.780	0.789	0.796	0.702	0.790	0.800	0.804

Notes: *p<0.1; **p<0.05; ***p<0.01. Sample restricted to non-presidential democracies and elections that produced a minority parliament, as detailed in Appendix C. All models are OLS, models (1)-(4) include party fixed effects and models (5)-(8) include party and survey fixed effects. Standard errors are clustered by survey in all models.

the premiership (except for the logit models of Table 7). The estimated marginal effect from the OLS models is that this additional probability (over and above being a core party) is now over 20% (model (4) of Table 7) and as high as 36.1% in model (8) of Table 8. On the basis of these patterns, a tentative conclusion is that when it comes to government participation the relevant measure seems to be the more inclusive core status of a party, whereas when it comes to control of the premiership, the most relevant measure is the more discerning Condorcet measure.

Among other variables in these models, the Seat plurality variable stands out both because of the consistent statistically significant finding across models and because of the substantial size of the estimated marginal effect. It is important to note here that seat plurality is not perfectly predictive of control of the premiership. Of the 104 elections in our sample, there were 18 elections in which the party that won the premiership position did not have a plurality of seats in parliament. The estimated effects for the remaining variables all have the expected signs but now both location variables and incumbency are not statistically significant in the fixed effect models of Table 8.

These analyses successfully pass a number of additional robustness checks including: Re-estimating the models in the subset of surveys where parties included in the survey receive more than 90% of the seats (Table C.3 of Appendix C and Appendix D); replicating the analysis in the sample that would be used for estimation in a fixed-effects logit model as recommended by Beck (2020) (available upon request); removing a possible highly collinear regressor in the fixed effects models (Appendix E); using the Manifesto RILE scores to measure the median and parties' distance from the median (available upon request).

5 Conclusions

We have developed a statistical procedure to test for core and Condorcet parties using readily available data from survey results. Perhaps contrary to the intuition from classic social choice theory discourse, we failed to reject the null hypothesis of a core party in close to half of the core tests we conducted in our dataset. We also failed to reject the Condorcet party null hypothesis for one party in 83 out of 196 election surveys in our sample. The resulting measures correlate sensibly with party observables that are theoretically related to core and Condorcet status, yet these measures add independent information compared to these observables. We used these measures to evaluate the (normative and theoretical) expectation that core and Condorcet parties are more likely to govern and found that core parties are substantially and statistically significantly more likely to participate in post-election governments, whereas the more discriminating (and therefore more reliably in the core) subset of Condorcet parties do not have a statistically significantly higher probability of participating in government over and above that of core parties. We also found that both types of parties have a significantly higher probability of heading the cabinet, with Condorcet parties having a statistically and substantively significantly higher probability of becoming the prime-minster's party over mere core parties. These findings can be thought of as positive news for scholars of representative democracy who espouse the normative criterion that core and Condorcet parties ought to govern. Furthermore, because these test procedures can be performed on, and the resultant measures can be built from, standard survey questionnaires that can be administered prior to an election, our findings also suggest the exciting promise of a practical method to predict government composition ahead of the actual election.

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Supplemental Appendix

Table of Contents

A Summary of alternative test results

The following table presents the summary of the binary test results for all three test statistics (MAX, MMM, and QLR) and for the weighted version of these tests. These three are modal among a large family of admissible statistics and we direct the reader to Romano, Shaikh and Wolf (2014) for details. Note that both the MMM and MAX versions are valid under minimal assumptions, whereas the QLR statistic requires an additional non-singularity condition on the covariance matrix of the moments, which may not be verified in finite samples. Note that the number of tests in the MMM and QLR versions are slightly smaller than the number of tests using the MAX test statistic. The former are numerically unstable and fail to compute in a few cases (specifically for the 2002 Brazilian election). Execution of the unweighted QLR test fails for 5 out of 6 parties in that election, as a result of a singular variance covariance matrix. The MMM and weighted QLR test statistics fail to compute for 2 parties in that election. The distribution of core and Condorcet parties is very similar across all tests.

Table A.1: Summary of test results

Test:	MMM	MMM weight	QLR	QLR weight	MAX	MAX weight
Total surveys	196	196	196	196	196	196
Total tests	1174	1176	1171	1174	1176	1176
% Core	48.722	49.065	48.506	48.807	48.129	48.469
Avg. core p-value	0.432	0.435	0.423	0.425	0.422	0.426
% Condorcet	7.836	7.908	7.259	7.24	7.058	7.228
Avg. Condorcet p-value	0.06	0.057	0.057	0.055	0.056	0.055
# Surveys with Core	196	196	196	196	196	196
# Surveys with Condorcet	89	91	83	84	83	84

Notes. All binary tests carried out using the two-step procedure of Romano, Shaikh and Wolf (2014), $\alpha = 0.05, \beta = 0.005, B = 20,000$. All p-values computed with $\beta = \alpha/10, B = 20,000$.

B Summary of results by country

In Table B.2, we disaggregate summary statistics by country for the test results and p-values using the MAX statistic. For each country in our analysis, the first and third columns of Table B.2 provide the average share of parties in a survey that pass the binary core and Condorcet winner test respectively, as well as their standard deviations across surveys. The second and fourth columns of the same table include the corresponding average core and Condorcet winner p-values respectively, and their standard deviations across surveys. The last two columns indicate the average number of parties in a test, and the number of surveys from that country in our analysis. There is significant variation between countries in the share of parties that pass the binary core tests, ranging from 1/9 parties in the solitary Albanian election, to all 9 parties in the solitary Croatian election.

Table B.2: Country aggregates

Country	Avg. share of core parties	Avg. core p-value	Avg. share of CWin parties	Avg. CWin p-value	Avg. # of parties	# surveys	Country	Avg. share of core parties	Avg. core p-value	Avg. share of CWin parties	Avg. CWin p-value	Avg. # of parties	# surveys
ALB	0.11 (–)	0.11 (–)	0.11 (–)	0.11 (–)	9.00 (–)	1	ARG	0.50 (–)	0.43 (–)	0.00 (–)	0.00 (–)	6.00 (–)	1
AUS	0.63 (0.14)	0.57 (0.11)	0.07 (0.10)	0.03 (0.03)	4.80 (0.84)	5	AUT	0.34 (0.21)	0.34 (0.20)	0.06 (0.10)	0.06 (0.10)	6.67 (0.58)	3
BELF	0.72 (0.16)	0.72 (0.16)	0.00 (0.00)	0.00 (0.00)	5.50 (0.71)	2	BELW	0.68 (0.11)	0.63 (0.16)	0.00 (0.00)	0.00 (0.00)	4.50 (0.71)	2
BGR	0.38 (0.34)	0.26 (0.16)	0.07 (0.10)	0.07 (0.10)	7.50 (0.71)	2	BLR	1.00 (0.00)	1.00 (0.00)	0.00 (0.00)	0.00 (0.00)	2.50 (0.71)	2
BRA	0.67 (0.45)	0.63 (0.42)	0.07 (0.10)	0.08 (0.10)	4.40 (1.82)	5	CAN	0.61 (0.24)	0.54 (0.23)	0.09 (0.12)	0.08 (0.10)	4.40 (0.55)	5

(Table continued on next page)

Country	Avg. share of core parties	Avg. core p-value	Avg. share of CWin parties	Avg. CWin p-value	Avg. # of parties	# surveys	Country	Avg. share of core parties	Avg. core p-value	Avg. share of CWin parties	Avg. CWin p-value	Avg. # of parties	# surveys
CHE	0.76 (0.11)	0.69 (0.17)	0.00 (0.00)	0.00 (0.00)	5.50 (1.00)	4	CHL	0.83 (0.14)	0.80 (0.15)	0.08 (0.17)	0.01 (0.01)	5.75 (2.06)	4
CZE	0.52 (0.21)	0.44 (0.19)	0.04 (0.09)	0.04 (0.09)	6.00 (1.00)	5	DEU	0.36 (0.24)	0.29 (0.22)	0.08 (0.08)	0.07 (0.07)	7.00 (0.82)	7
DNK	0.26 (0.12)	0.26 (0.12)	0.04 (0.07)	0.04 (0.07)	7.67 (1.53)	3	ESP	0.33 (0.11)	0.28 (0.10)	0.17 (0.14)	0.18 (0.13)	4.75 (1.26)	4
EST	0.50 (–)	0.43 (–)	0.17 (–)	0.03 (–)	6.00 (–)	1	FIN	0.42 (0.14)	0.37 (0.12)	0.06 (0.07)	0.02 (0.02)	7.75 (0.50)	4
FRA	0.42 (0.30)	0.42 (0.30)	0.11 (0.07)	0.06 (0.08)	6.75 (0.96)	4	GBR	0.71 (0.08)	0.56 (0.17)	0.11 (0.19)	0.11 (0.19)	3.67 (1.15)	3
GRC	0.44 (0.33)	0.42 (0.31)	0.08 (0.09)	0.08 (0.09)	7.00 (0.82)	4	HKG	0.80 (0.19)	0.61 (0.23)	0.03 (0.08)	0.02 (0.03)	5.67 (1.51)	6
HRV	1.00 (–)	0.91 (–)	0.00 (–)	0.00 (–)	9.00 (–)	1	HUN	0.61 (0.35)	0.51 (0.37)	0.06 (0.10)	0.05 (0.09)	5.67 (0.58)	3
IRL	0.35 (0.17)	0.33 (0.14)	0.08 (0.10)	0.05 (0.08)	6.25 (0.50)	4	ISL	0.42 (0.13)	0.35 (0.13)	0.05 (0.08)	0.05 (0.08)	6.14 (1.07)	7
ISR	0.52 (0.18)	0.42 (0.12)	0.08 (0.10)	0.04 (0.05)	5.75 (0.50)	4	ITA	0.75 (0.35)	0.69 (0.44)	0.00 (0.00)	0.00 (0.00)	6.00 (0.00)	2
JPN	0.31 (0.14)	0.31 (0.14)	0.07 (0.08)	0.03 (0.05)	6.25 (1.89)	4	KEN	0.50 (–)	0.44 (–)	0.00 (–)	0.00 (–)	6.00 (–)	1
KGZ	0.80 (–)	0.80 (–)	0.00 (–)	0.00 (–)	5.00 (–)	1	KOR	0.69 (0.25)	0.57 (0.24)	0.00 (0.00)	0.00 (0.00)	5.20 (1.30)	5
LTU	0.31 (0.27)	0.31 (0.27)	0.06 (0.09)	0.06 (0.09)	7.00 (1.41)	2	LVA	0.67 (0.17)	0.57 (0.24)	0.06 (0.10)	0.01 (0.01)	6.00 (0.00)	3
MEX	0.46 (0.24)	0.40 (0.20)	0.05 (0.08)	0.05 (0.08)	6.29 (1.11)	7	MNE	0.93 (0.10)	0.63 (0.41)	0.07 (0.10)	0.06 (0.08)	6.00 (1.41)	2
NLD	0.29 (0.20)	0.26 (0.16)	0.06 (0.07)	0.06 (0.07)	8.50 (1.00)	4	NOR	0.29 (0.15)	0.25 (0.15)	0.11 (0.06)	0.07 (0.05)	7.83 (1.17)	6
NZL	0.21 (0.10)	0.19 (0.09)	0.08 (0.07)	0.08 (0.06)	7.67 (1.03)	6	PER	0.66 (0.23)	0.60 (0.24)	0.03 (0.07)	0.01 (0.02)	5.80 (0.45)	5

(Table continued on next page)

Country	Avg. share of core parties	Avg. core p-value	Avg. share of CWin parties	Avg. CWin p-value	Avg. # of parties	# surveys	Country	Avg. share of core parties	Avg. core p-value	Avg. share of CWin parties	Avg. CWin p-value	Avg. # of parties	# surveys
PHL	0.86 (0.25)	0.81 (0.33)	0.05 (0.08)	0.03 (0.04)	6.33 (0.58)	3	POL	0.32 (0.12)	0.26 (0.06)	0.12 (0.11)	0.11 (0.11)	6.00 (1.58)	5
PRT	0.57 (0.33)	0.48 (0.25)	0.05 (0.10)	0.05 (0.10)	5.50 (0.58)	4	ROU	0.42 (0.08)	0.35 (0.10)	0.07 (0.09)	0.04 (0.06)	6.20 (0.45)	5
RUS	0.28 (0.19)	0.18 (0.02)	0.17 (0.00)	0.17 (0.00)	6.00 (0.00)	3	SRB	0.14 (−)	0.14 (−)	0.14 (−)	0.14 (−)	7.00 (−)	1
SVK	0.56 (0.27)	0.44 (0.42)	0.06 (0.09)	0.06 (0.09)	8.00 (0.00)	2	SVN	0.76 (0.05)	0.65 (0.11)	0.00 (0.00)	0.00 (0.00)	7.25 (0.96)	4
SWE	0.25 (0.13)	0.23 (0.12)	0.11 (0.08)	0.11 (0.08)	7.50 (1.73)	4	THA	0.33 (0.18)	0.32 (0.18)	0.27 (0.20)	0.23 (0.24)	5.00 (2.65)	3
TUR	0.26 (0.21)	0.26 (0.21)	0.15 (0.03)	0.10 (0.07)	7.00 (1.73)	3	TWN	0.68 (0.27)	0.64 (0.27)	0.03 (0.08)	0.02 (0.06)	3.83 (1.33)	6
UKR	0.83 (−)	0.83 (−)	0.00 (−)	0.00 (−)	6.00 (−)	1	URY	0.20 (−)	0.20 (−)	0.20 (−)	0.20 (−)	5.00 (−)	1
USA	0.73 (0.25)	0.73 (0.25)	0.27 (0.25)	0.21 (0.26)	2.20 (0.45)	5	ZAF	0.17 (0.00)	0.17 (0.00)	0.17 (0.00)	0.17 (0.00)	6.00 (0.00)	2

Notes. Standard errors in parentheses. All binary tests carried out using the two-step procedure of Romano, Shaikh and Wolf (2014), with the MAX statistic, $\alpha = 0.05$, $\beta = 0.005, B = 20,000$. All p-values computed using the two-step procedure of Romano, Shaikh and Wolf (2014) with the MAX statistic, $\beta = \alpha/10, B = 20,000$.

C Data exclusions and variable construction

C.1 Data exclusions & Coverage

As we briefly explain at the beginning of Section 4 and we further document here, the following data are excluded from the regression analyses in that section:

- Presidential systems, notably surveys from Argentina, Brazil, Chile, Kenya, South Korea, Mexico, Peru, the Philippines, Uruguay, and the United States of America.[29]
- Autocracies or anocracies according to the Polity IV criteria (Marshall and Gurr 2020) for that survey year, that is Belarus, Kyrgyzstan, Thailand 2007, and Turkey 2015, 2018. Hong Kong is excluded for similar reasons (though not covered by Polity).
- Taiwan 1996, Ukraine 1998, Russia 1999, and Thailand 2001 because they are not covered in our sources on government participation.
- France 2002, France 2007, France 2012, France 2017, Lithuania 1997, Romania 2009, 2014, Russia 2004, and Taiwan 2004, 2008 because these surveys coincided with presidential elections rather than parliamentary elections.
- Similarly, Japan 2004 and 2007 because these surveys coincided with upper house elections rather than lower house elections.
- Belgium because the surveys are conducted at the subnational level (separately for Flanders and Wallonia).

[29] Our findings for the correlation of party-specific variables with the core and Condorcet p-values are robust to the inclusion of presidential systems survey data for the subset of presidential system surveys that coincided with a lower house election. These results are available upon request.

- Japan 1996 and Taiwan 2012, 2016 because CSES did not ask respondents to provide a left-right score for the parties.
- One of the two 2002 German election surveys, opting for the telephone survey of that year.

In addition to the above exclusions, in the analysis in the second half of section 4 focusing on government participation and control of the prime-minister position, we exclude surveys for elections that produced a seat majority party. These are:

- Australia 1996, 2007, 2013, Canada 1997, 2011, 2015, Spain 2000, United Kingdom 1997, 2005, 2015, Greece 2009, Hungary 2018, Portugal 2005, Thailand 2011, Turkey 2011, South Africa, 2009, 2014.

Both sets of surveys included in the analysis of section 4 were timed around parliamentary elections to the lower House and we can therefore report the total number of seats received after the election by the parties included in these CSES surveys (and in our core and Condorcet tests). We report these seat fractions in Table C.3. Evidently, more than half of these surveys effectively include all parties that earn seats (more than 99%). The fraction of the surveys with parties exceeding 95% of total seats is over 80%, and this fraction reaches 90% of surveys with parties that earn at least 90% of the seats in parliament.

Table C.3: Total fraction of seats earned by parties by survey/election

	$\geq 99\%$	$(99\%, 95\%]$	$(95\%, 90\%]$	$< 90\%$	TOTAL
Surveys in Table 4	65	34	10	12	121
	53.7%	28.1%	8.3%	9.9%	100%
Surveys in Tables 5, 6, 7, and 8	61	23	9	11	104
	58.7%	22.1%	8.7%	10.6%	100%

Notes. Total fraction of seats earned by parties included in the CSES surveys used in the analyses of Section 4.

C.2 Median calculation

Because the sum of vote (seat) shares of parties in our sample need not amount to 100%, in the main body we calculate the median (and derivative variables) as follows: We first calculate the 'left' median, by putting all the missing vote (seat) share to the left of the most leftward party in the election that we have in the dataset. We calculate a 'right' median in a similar way. In many cases the left and right medians coincide and then we have a unique median candidate to which we assign a value of 1 (and zero to all others). If the left and right medians do not coincide, then all parties between the left and the right median are assigned $\frac{1}{n}$, where n is the number of parties in that interval, and we assign 0 to all other parties. For the purposes of calculating distance from the median in the latter cases, we calculate an expected median as the average across the n possible medians.

A simpler alternative to the above calculation is to compute the median party of the available parties instead. In essence, this implicitly assumes that the missing vote or seat share is split equally to the left of the left-most party and to the right of the right-most party. Note that the median party calculated in this fashion has to also be one of the medians calculated using the other method. All results from Tables 4-8 are robust to the use of this simpler median.

D Restricting the sample to surveys with $\geq 90\%$ seat share coverage

As outlined in Appendix C, our sample in the regression results we present in the main text includes surveys that do not include all parties than win representation in the parliament. A potential concern for these surveys is that our measures of core and Condorcet parties are computed using an incomplete set of moment inequalities, as we cannot compare all available parties in the election. Though some of the missing seat share may correspond to independent candidates, when it corresponds to missing parties, these earn a sizable share of seats in parliament and, therefore, a sizable vote share. Obviously it is not possible for us to retroactively include these parties in these surveys. The fact that we include survey (and party) fixed effects in these regressions mitigates (at least partially) for any bias from these omitted parties.

We further investigate whether our results in Tables 4-8 are driven by the inclusion of these surveys with low seat share coverage. Specifically, we re-estimate these models restricting the sample to surveys for which we have 90% or higher seat coverage, effectively reducing the sample to 109 surveys and 688 party-survey pairs from the sample used in Table 4, and to 93 surveys and 620 party-survey pairs from the sample used in Table 5-8 and report the results in this appendix. Our findings from Tables 4-8 are robust to this sample restriction.

Table D.4: Test results and other observables (surveys with $\geq$ 90% seat share coverage)

	Dependent variable:					
	Core p-value	Condorcet p-value	Core p-value	Condorcet p-value	Core p-value	Condorcet p-value
	(1)	(2)	(3)	(4)	(5)	(6)
LH vote	0.019***	0.005***	0.021***	0.010***	0.024***	0.012***
	(0.002)	(0.001)	(0.004)	(0.003)	(0.004)	(0.003)
Electoral Plurality	0.159**	0.212***	0.164**	0.143***	0.134**	0.134***
	(0.062)	(0.046)	(0.078)	(0.052)	(0.056)	(0.040)
Electoral Median	0.108**	0.146***	0.053	0.060	0.060	0.074
	(0.054)	(0.044)	(0.079)	(0.069)	(0.056)	(0.052)
Distance from median	−0.035**	−0.002	−0.053*	−0.016	−0.063***	−0.014
	(0.014)	(0.006)	(0.029)	(0.014)	(0.018)	(0.014)
Incumb. pty.	0.001	−0.038**	−0.107**	−0.027	−0.090**	−0.036
	(0.038)	(0.019)	(0.049)	(0.031)	(0.037)	(0.026)
Constant	0.114***	−0.050***				
	(0.039)	(0.015)				
Observations	688	688	688	688	688	688
R^2	0.399	0.366	0.728	0.644	0.834	0.722

Notes: *p<0.1; **p<0.05; ***p<0.01. Models (1) and (2) are OLS, models (3) and (4) include party fixed effects, and models (5) and (6) include party and survey fixed effects. Standard errors are clustered by survey in all models. All models restrict the analysis to observations from surveys with at least 90% seat share coverage within the sample used in Table 4.

Table D.5: Government participation (surveys with $\geq$ 90% seat share coverage)

	OLS				*Logistic*			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Core p-value	0.404***	0.234***		0.233***	1.792***	1.142***		1.117***
	(0.043)	(0.052)		(0.051)	(0.205)	(0.250)		(0.245)
Condorcet p-value	0.350***		0.040	0.028	2.421***		1.292	0.993
	(0.060)		(0.080)	(0.078)	(0.789)		(1.190)	(1.026)
LH seats		0.003	0.007***	0.003		0.021*	0.040***	0.019
		(0.002)	(0.002)	(0.002)		(0.012)	(0.011)	(0.013)
Seat Plurality		0.228***	0.258***	0.221**		1.283**	1.265**	1.163**
		(0.080)	(0.089)	(0.086)		(0.508)	(0.515)	(0.531)
Legislative Median		0.155**	0.158**	0.153**		0.699*	0.707*	0.688*
		(0.062)	(0.064)	(0.062)		(0.385)	(0.378)	(0.386)
Distance to median		−0.067***	−0.077***	−0.066***		−0.497***	−0.533***	−0.491***
		(0.013)	(0.013)	(0.013)		(0.114)	(0.112)	(0.115)
Incumbent party		0.117**	0.121**	0.118**		0.662**	0.662**	0.672**
		(0.054)	(0.056)	(0.054)		(0.310)	(0.306)	(0.311)
Constant	0.202***	0.264***	0.304***	0.265***	−1.363***	−1.037***	−0.842***	−1.025***
	(0.020)	(0.038)	(0.040)	(0.038)	(0.123)	(0.237)	(0.233)	(0.235)
Observations	620	620	620	620	620	620	620	620
R^2	0.218	0.329	0.299	0.329				
Log Likelihood					−338.687	−294.037	−302.932	−293.250

Notes: *p<0.1; **p<0.05; ***p<0.01. Models (1)-(4) are OLS and models (5)-(8) are logistic. Standard errors are clustered by survey in all models. All models restrict the analysis to observations from surveys with at least 90% seat share coverage within the sample used in Table 5.

Table D.6: Government participation (including fixed effects, surveys with $\geq$ 90% seat share coverage)

	Party fixed effects				*Party and survey fixed effects*			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Core p-value	0.381***	0.167**		0.170**	0.450***	0.209***		0.200**
	(0.085)	(0.080)		(0.081)	(0.084)	(0.077)		(0.079)
Condorcet p-value	0.484***		0.109	0.124	0.440***		0.144	0.094
	(0.114)		(0.142)	(0.137)	(0.108)		(0.118)	(0.118)
LH seats		0.011***	0.013***	0.010**		0.012***	0.016***	0.011***
		(0.004)	(0.004)	(0.004)		(0.004)	(0.004)	(0.004)
Seat Plurality		0.301***	0.312***	0.281***		0.275***	0.285***	0.263***
		(0.098)	(0.109)	(0.105)		(0.073)	(0.082)	(0.079)
Legislative Median		0.093	0.071	0.084		0.080	0.059	0.074
		(0.101)	(0.105)	(0.102)		(0.076)	(0.079)	(0.076)
Distance to median		−0.106***	−0.116***	−0.104***		−0.112***	−0.122***	−0.111***
		(0.034)	(0.037)	(0.034)		(0.026)	(0.029)	(0.026)
Incumbent party		−0.138	−0.151	−0.137		−0.116*	−0.128*	−0.116*
		(0.093)	(0.094)	(0.092)		(0.070)	(0.070)	(0.069)
Observations	620	620	620	620	620	620	620	620
R^2	0.632	0.697	0.692	0.698	0.672	0.733	0.727	0.733

Notes: *p<0.1; **p<0.05; ***p<0.01. All models are OLS, models (1)-(4) include party fixed effects and models (5)-(8) include party and survey fixed effects. Standard errors are clustered by survey in all models. All models restrict the analysis to observations from surveys with at least 90% seat share coverage within the sample used in Table 6.

Table D.7: Premiership (surveys with $\geq$ 90% seat share coverage)

	OLS				*Logistic*			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Core p-value	0.260***	0.077***		0.074***	3.460***	1.818***		1.766***
	(0.022)	(0.025)		(0.024)	(0.433)	(0.599)		(0.548)
Condorcet p-value	0.626***		0.228**	0.224**	2.973***		0.919	0.769
	(0.066)		(0.093)	(0.092)	(0.545)		(0.896)	(0.817)
LH seats		0.004***	0.005***	0.004**		0.091***	0.107***	0.085***
		(0.001)	(0.002)	(0.002)		(0.017)	(0.017)	(0.019)
Seat Plurality		0.505***	0.464***	0.451***		1.992***	2.111***	1.900***
		(0.081)	(0.090)	(0.090)		(0.543)	(0.568)	(0.580)
Legislative Median		−0.011	−0.033	−0.034		−0.736	−0.923	−0.835
		(0.038)	(0.037)	(0.037)		(0.578)	(0.630)	(0.592)
Distance to median		−0.016**	−0.019***	−0.016**		−0.400**	−0.492***	−0.378*
		(0.007)	(0.007)	(0.007)		(0.197)	(0.189)	(0.203)
Incumbent premier		0.061	0.059	0.064		0.304	0.312	0.329
		(0.063)	(0.062)	(0.061)		(0.554)	(0.563)	(0.574)
Constant	0.006	−0.010	0.010	−0.002	−4.419***	−5.196***	−4.283***	−5.079***
	(0.005)	(0.015)	(0.014)	(0.014)	(0.410)	(0.621)	(0.357)	(0.546)
Observations	620	620	620	620	620	620	620	620
R^2	0.382	0.573	0.580	0.586				
Log Likelihood					−149.467	−100.581	−105.195	−99.899

Notes: *p<0.1; **p<0.05; ***p<0.01. Models (1)-(4) are OLS and models (5)-(8) are logistic. Standard errors are clustered by survey in all models. All models restrict the analysis to observations from surveys with at least 90% seat share coverage within the sample used in Table 7.

Table D.8: Premiership (including fixed effects, surveys with $\geq$ 90% seat share coverage)

	Party fixed effects				*Party and survey fixed effects*			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Core p-value	0.274***	0.110***		0.119***	0.297***	0.148***		0.113***
	(0.048)	(0.041)		(0.042)	(0.047)	(0.046)		(0.042)
Condorcet p-value	0.637***		0.329**	0.339***	0.650***		0.389***	0.360***
	(0.106)		(0.132)	(0.130)	(0.088)		(0.110)	(0.106)
LH seats		0.007**	0.007**	0.005		0.008***	0.008***	0.006**
		(0.003)	(0.003)	(0.003)		(0.003)	(0.003)	(0.003)
Seat Plurality		0.553***	0.520***	0.499***		0.538***	0.502***	0.490***
		(0.106)	(0.115)	(0.112)		(0.080)	(0.088)	(0.085)
Legislative Median		−0.006	−0.038	−0.029		−0.007	−0.038	−0.030
		(0.060)	(0.064)	(0.062)		(0.048)	(0.052)	(0.050)
Distance to median		−0.033**	−0.035**	−0.027*		−0.032**	−0.034**	−0.027**
		(0.016)	(0.017)	(0.016)		(0.014)	(0.014)	(0.014)
Incumbent premier		−0.069	−0.083	−0.073		−0.074	−0.089	−0.079
		(0.089)	(0.085)	(0.085)		(0.067)	(0.063)	(0.064)
Observations	620	620	620	620	620	620	620	620
R^2	0.712	0.788	0.797	0.804	0.723	0.796	0.807	0.810

Notes: *p<0.1; **p<0.05; ***p<0.01. All models are OLS, models (1)-(4) include party fixed effects and models (5)-(8) include party and survey fixed effects. Standard errors are clustered by survey in all models. All models restrict the analysis to observations from surveys with at least 90% seat share coverage within the sample used in Table 8.

E Multicollinearity

A possible concern for some of our null findings, especially in our government inclusion and premiership models is that several of our independent variables may be collinear. In model specifications without fixed effects, no such collinearity problem is discernible with standard diagnostics. On the other hand, in model specifications with party fixed effects, the Seat share variable is identified as a likely highly collinear regressor (the VIF statistic for lower house seat is larger than thirteen). Though there is no clear resolution for this problem, other than increasing sample size, we re-estimate several of the main models dropping this regressor. We present these results in Table E.9. As is evident, the results from the main LPM models of government inclusion (models (1) and (2) analogous to models (4) and (8) of Table 6), and from corresponding models for control of the premiership (models (3) and (4) analogous to models (4) and (8) of Table 8) are all robust after removing the LH seats variable.

Table E.9: Specifications removing lower house seat share

	Dependent variable:			
	Government inclusion		Premiership	
	(1)	(2)	(3)	(4)
Core p-value	0.235***	0.289***	0.142***	0.150***
	(0.078)	(0.075)	(0.043)	(0.042)
Condorcet p-value	0.175	0.146	0.355***	0.380***
	(0.124)	(0.111)	(0.123)	(0.102)
Seat Plurality	0.392***	0.374***	0.574***	0.566***
	(0.100)	(0.078)	(0.099)	(0.076)
Legislative Median	0.088	0.077	−0.027	−0.028
	(0.101)	(0.075)	(0.057)	(0.046)
Distance to median	−0.101***	−0.107***	−0.023	−0.024
	(0.030)	(0.023)	(0.017)	(0.015)
Incumbent party	−0.137	−0.119*		
	(0.084)	(0.065)		
Incumbent premier			−0.065	−0.070
			(0.078)	(0.061)
Observations	680	680	680	680
R^2	0.684	0.721	0.794	0.802

Notes: *p<0.1; **p<0.05; ***p<0.01. Models (1) and (3) are OLS with party fixed effects, and models (2) and (4) are OLS with party and survey fixed effects. Standard errors are clustered by survey in all models.