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9 Asymmetric Information

9.1 Introduction

In some bargaining situations at least one of the players knows something of relevance that the other player does not. For example, when bargaining over the price of her second-hand car the seller knows its quality but the buyer does not. In such a bargaining situation, the seller has private information; and there exists an asymmetry in information between the players. In this chapter I study the role of asymmetric information on the bargaining outcome.

A player may in general have private information about a variety of things that may be relevant for the bargaining outcome, such as her preferences, outside option and inside option. However, in order to develop the main fundamental insights in a simple manner attention is focused on the following archetypal bargaining situation. A seller and a buyer are bargaining over the price at which to trade an indivisible object (such as a second-hand car, or a unit of labour). The payoff to each player (from trading) depends on the agreed price and on her reservation value.[1] A key assumption is that at least one player's reservation value is her private information.

I begin the study of this bargaining situation by addressing the *normative*

[1] The buyer's reservation value is the maximum price at which she is willing to buy. Symmetrically, the seller's reservation value is the minimum price at which she is willing to sell.

question of whether or not the bargaining outcome can be *ex-post* efficient.[2] Let me make this question a bit more precise. A bargaining procedure combined with the players' information and preferences in the bargaining situation under consideration defines a bargaining game with imperfect information. I shall say that a procedure *induces* a bargaining game. If there exists a bargaining procedure — no matter how implausible or plausible it may be — such that the induced bargaining game has a Bayesian Nash Equilibrium (BNE) that generates an *ex-post* efficient outcome, then the bargaining outcome can be *ex-post* efficient. On the other hand, if for *any* bargaining procedure *all* of the BNE of the induced bargaining game generate *ex-post* inefficient outcomes, then the bargaining outcome cannot be *ex-post* efficient.

The following argument illustrates the possibility that the bargaining outcome cannot be *ex-post* efficient. A buyer and a seller are bargaining over the price of a second-hand car, whose quality is the seller's private information. If she owns a low quality car, then she has an incentive to pretend to own a high quality car in order to obtain a relatively high price. Since the buyer is aware of this 'incentive to lie', the maximum price that she might be willing to pay may be strictly less than the high reservation value of a seller owning a high quality car. Thus, if the seller actually owns a high quality car, then mutually beneficial trade between the two parties may fail to occur.[3]

Section 9.2 addresses the normative question stated above when exactly one player's reservation value is her private information. The answer to the normative question depends on whether or not the players' reservation values are independent of each other. When the players' reservation values are independent of each other, I say that the values are *private*; otherwise, they are *correlated*. If the players' reservation values are private, then the bargaining outcome can be *ex-post* efficient. But if the reservation values are correlated, then (under a fairly general condition) the bargaining outcome cannot be *ex-post* efficient. Two applications are studied in Section 9.3.

[2]The bargaining outcome is *ex-post* efficient if and only if after all of the information is revealed the players' payoffs associated with the bargaining outcome are Pareto-efficient. The concept of *ex-post* efficiency is also known as full-information efficiency.

[3]In his classic paper, Akerlof (1970), George Akerlof put forward this type of argument, but in the context of competitive markets (with asymmetric information).

One application concerns the normative question of whether or not it is possible for a firm and its unionized workforce to reach an agreement (over the wage rate) without any costly delay. The other application concerns the normative question of whether or not it is possible for a plantiff and a defendant to settle their dispute out-of-court.

Section 9.4 addresses the normative question when each player's reservation is her private information. A main result obtained here is that (under some fairly general conditions) the bargaining outcome cannot be *ex-post* efficient, whether or not the players' reservation values are independent of each other. Section 9.5 extends the two applications studied in Section 9.3.

Section 9.6 addresses some *positive* questions in the context of a bargaining model in which exactly one player has private information, and the other player makes repeated offers. The two main questions that motivate the study of this model are as follows. Firstly, to what extent is the equilibrium payoff of the player who makes all the offers adversely affected when her opponent has private information? And secondly, under what conditions (if any) is the privately held information revealed through time via the sequence of equilibrium offers? It will be shown that the answers to these questions depend, in particular, on (i) whether or not offers are retractable, (ii) whether or not the costs to the players of haggling are small, and (iii) whether or not gains from trade are strictly positive with probability one. An application to bargaining over a *menu* of wage-quality contracts is studied in Section 9.7.

9.2 Efficiency under One-Sided Uncertainty

I study a bargaining situation in which player S owns (or, can produce) an indivisible object that player B wants to buy. If agreement is reached to trade at price p ($p \geq 0$), then the payoffs to the seller (player S) and the buyer (player B) are respectively $p - c$ and $v - p$, where c denotes the seller's reservation value (or, cost of production) and v denotes the buyer's reservation value (or, the maximum price at which she is willing to buy). If the players do not reach an agreement to trade, then each player's payoff is zero.

A key assumption is that exactly one player's reservation value is her private information. Section 9.2.1 studies the case in which the players' reservation values are independent of each other, while Section 9.2.2 studies the case in which the players' reservation values are correlated.

The outcome of this bargaining situation is *ex-post* efficient if and only if when $v \geq c$ the players reach an agreement to trade, and when $v < c$ the players do not reach an agreement to trade.

My objective is to address the normative question of whether or not the outcome of the bargaining situation described above can be *ex-post* efficient. As mentioned in Section 9.1, if there exists a bargaining procedure such that the induced bargaining game has a BNE that generates an *ex-post* efficient outcome, then the bargaining outcome can be *ex-post* efficient. On the other hand, if for *any* bargaining procedure *all* of the BNE of the induced bargaining game generate *ex-post* inefficient outcomes, then the bargaining outcome cannot be *ex-post* efficient.

9.2.1 The Case of Private Values

In this section it is assumed that the players' reservation values are independent of each other, and exactly one player has private information about her reservation value. The other player's reservation value is known to both players.

I begin by studying the case in which the buyer's reservation value is her private information. This asymmetry in information is modelled as follows. The buyer's reservation value is a random draw from the following (binary) probability distribution: with probability α (where $0 < \alpha < 1$) the buyer's reservation value is H, and with probability $1 - \alpha$ the buyer's reservation value is L, where $H > L$. The buyer knows the realization of the random draw, but the seller does not. The seller only knows that the buyer's reservation value is a random draw from this probability distribution. The following lemma establishes that the bargaining outcome can be *ex-post* efficient.[4]

Lemma 9.1. *There exists a bargaining procedure such that the induced bargaining game has an ex-post efficient BNE.*

[4]The lemma is valid whatever is the magnitude of c relative to H and L. It should be noted that if $c > H$ then gains from trade do not exist, but if $L \geq c$ then gains from trade exist with probability one. And if $H \geq c > L$ then with probability α they exist, but with probability $1 - \alpha$ they do not.

Proof. Consider the following bargaining procedure. The buyer makes an offer to the seller. If she accepts the offer, then agreement is struck and the game ends. But if she rejects the offer, then the game ends with no agreement. Letting p_H^* and p_L^* respectively denote the buyer's price offers when $v = H$ and $v = L$, the following pair of strategies is a BNE: $p_H^* = \min\{H, c\}$, $p_L^* = \min\{L, c\}$, and the seller accepts a price offer p if and only if $p \geq c$. The lemma follows immediately, because this BNE is *ex-post* efficient. □

It is now shown that Lemma 9.1 is also valid when the buyer's reservation value can take more than two possible numbers. Let F_B denote the cumulative probability distribution from which the buyer's reservation value is randomly drawn, where the support of F_B contains two or more numbers. The proof of Lemma 9.1 is still valid, but with the following modification to the buyer's equilibrium strategy: $p^*(v) = \min\{v, c\}$ for all v, where $p^*(v)$ denotes the buyer's price offer when her reservation value is v.

I now study the case in which the seller's reservation value is her private information, and the buyer's reservation value is known to both players. As above, I model this asymmetry in information by considering the seller's reservation value to be a random draw from a probability distribution F_S — whose support contains two or more numbers — the realisation of which is only revealed to the seller. Consider the following bargaining procedure. The seller makes an offer to the buyer. If she accepts the offer, then agreement is struck and the game ends. But if she rejects the offer, then the game ends with no agreement. The following pair of strategies is a BNE of the induced bargaining game: $p^*(c) = \max\{v, c\}$ for all c, where $p^*(c)$ denotes the seller's price offer when her reservation value is c, and the buyer accepts a price offer p if and only if $p \leq v$. Lemma 9.1 follows immediately, because this BNE is *ex-post* efficient.

The following proposition summarizes the results obtained above.

Proposition 9.1 (One-Sided Uncertainty with Private Values). *If the players' reservation values are independent of each other, and exactly one player's reservation value is her private information, then the bargaining outcome can be ex-post efficient.*

9.2.2 The Case of Correlated Values

In this section I assume that the player's reservation values are correlated, and exactly one player has private information about her reservation value. This assumption is modelled as follows. There is a parameter θ — which is a real number — that determines both players' reservation values, and furthermore, the value of θ is the private information of exactly one player. It is assumed that each player's reservation value is strictly increasing in θ. Furthermore, for any θ, the buyer's reservation value — which is denoted by $v(\theta)$ — is greater than or equal to the seller's reservation value — which is denoted by $c(\theta)$.[5]

I begin by considering the case in which it is the seller who has private information about θ. This asymmetry in information is modelled as follows. The value of θ is a random draw from the following (binary) probability distribution: with probability α (where $0 < \alpha < 1$) the value of θ is H, and with probability $1 - \alpha$ the value of θ is L, where $H > L$. The seller knows the realization of the random draw, but the buyer does not. The buyer only knows that the value of θ is a random draw from this probability distribution. The following lemma establishes that the bargaining outcome can be *ex-post* efficient if and only if $v^e \geq c(H)$, where $v^e = \alpha v(H) + (1 - \alpha)v(L)$ is the buyer's expected reservation value.

Lemma 9.2. *(a) If $v^e \geq c(H)$, where $v^e = \alpha v(H) + (1-\alpha)v(L)$, then there exists a bargaining procedure such that the induced bargaining game has an ex-post efficient BNE.*
(b) If $v^e < c(H)$, then for any bargaining procedure the induced bargaining game does not have an ex-post efficient BNE.

Proof. I first establish Lemma 9.2(a). Consider the following bargaining procedure. The seller makes an offer to the buyer. If she accepts the offer, then agreement is struck and the game ends. But if she rejects the offer, then the game ends with no agreement. Since $v^e \geq c(H)$, the following pair of strategies is a BNE: $p^*(H) = p^*(L) = c(H)$ (where $p^*(H)$ and $p^*(L)$ are respectively the seller's price offers when $\theta = H$ and $\theta = L$), the buyer accepts the price $p = c(H)$ and rejects any price $p \neq c(H)$. The desired conclusion follows immediately, because this BNE is *ex-post* efficient. □

[5] This implies that the outcome of this bargaining situation is *ex-post* efficient if and only if the players reach an agreement to trade whatever value θ takes.

I now proceed to prove Lemma 9.2(b). In order to do so I need to consider the set of *all* possible bargaining procedures. However, I begin by considering a particular subset of the set of all bargaining procedures, which is called the set of all *direct revelation* procedures. In the context of the bargaining situation under consideration, a direct revelation procedure (DRP) is characterized by four numbers: λ_L, λ_H, p_L and p_H, where $\lambda_s \in [0, 1]$ and $p_s \geq 0$ ($s = L, H$). In a DRP the seller announces a possible value of θ. If s denotes the announced value (where $s \in \{L, H\}$), then with probability λ_s trade occurs at price p_s, and with probability $1 - \lambda_s$ trade does not occur.

Fix an arbitrary DRP, and consider the induced bargaining game (which is a single-person decision problem). Let $s(\theta) \in \{L, H\}$ denote the seller's announcement if the true (realized) value is θ ($\theta = L, H$). The DRP is *incentive-compatible* if and only if in the induced bargaining game the seller announces the truth — that is, $s^*(L) = L$ and $s^*(H) = H$. Thus, the DRP is incentive-compatible if and only if the following two inequalities are satisfied

$$\lambda_L(p_L - c(L)) \geq \lambda_H(p_H - c(L)) \tag{9.1}$$

$$\lambda_H(p_H - c(H)) \geq \lambda_L(p_L - c(H)). \tag{9.2}$$

Inequalities 9.1 and 9.2 are respectively known as the incentive-compatibility constraints for the *low-type* seller and *high-type* seller.[6] Inequality 9.1 states that the expected payoff to the low-type seller by announcing the truth is greater than or equal to her expected payoff by telling a lie. Similarly, inequality 9.2 states that the expected payoff to the high-type seller by announcing the truth is greater than or equal to her expected payoff by telling a lie.

An incentive-compatible DRP is *individually-rational* if and only if in the incentive-compatible DRP each type of seller and the buyer obtain an expected payoff that is not less than their respective payoff from disagreement (which equals zero). That is, if and only if the following three inequalities

[6]The seller is said to be of 'low-type' (respectively, 'high-type') if the true (realized) value of θ is L (respectively, H).

are satisfied

$$\lambda_L(p_L - c(L)) \geq 0 \tag{9.3}$$

$$\lambda_H(p_H - c(H)) \geq 0 \tag{9.4}$$

$$\alpha\lambda_H(v(H) - p_H) + (1-\alpha)\lambda_L(v(L) - p_L) \geq 0. \tag{9.5}$$

Since (by assumption) $v(H) \geq c(H)$ and $v(L) \geq c(L)$, a DRP is *ex-post efficient* if and only if the buyer trades with the seller of either type with probability one. That is, if and only if

$$\lambda_L = \lambda_H = 1. \tag{9.6}$$

I now state a rather powerful result — which is known as the Revelation Principle — that allows me to establish Lemma 9.2(b) by considering only the set of all incentive-compatible and individually-rational direct revelation procedures.

Theorem 9.1 (The Revelation Principle). *Fix an arbitrary bargaining situation with asymmetric information and an arbitrary bargaining procedure. For any BNE outcome of the induced bargaining game there exists an incentive-compatible and individually-rational DRP that implements the BNE outcome.*

Proof. This result is proven and discussed in the context of bargaining situations in Myerson (1979). The proof can also be found in most advanced microeconomics and game theory texts — see, for example, Kreps (1990a), Fudenberg and Tirole (1991), Gibbons (1992) and Mas-Colell, Whinston and Green (1995). □

It follows from the Revelation Principle that if there does not exist an incentive-compatible and individually-rational DRP that is *ex-post* efficient, then there does not exist a bargaining procedure whose induced bargaining game has an *ex-post* efficient BNE. Lemma 9.2(b) is therefore an immediate consequence of the following claim.

Claim 9.1. *If $v^c < c(H)$, where v^c is defined in Lemma 9.2, then there does not exist an incentive-compatible and individually-rational DRP that is ex-post efficient.*

Proof. Suppose, to the contrary, that there exists a DRP that satisfies (9.1)–(9.6). Substituting (9.6) into (9.1) and (9.2), it follows that $p_L = p_H$. Hence, after substituting (9.6) into (9.3)–(9.5), it follows from (9.3)–(9.5) that $v^e \geq c(H)$, thus contradicting the hypothesis. □

A Generalization to More Than Two Types

The results obtained above may be generalized quite easily to the case when θ takes more than two possible values. Assume that θ is a random draw from a probability distribution G — whose support J contains two or more numbers — the realization of which is only revealed to the seller. Let $\underline{\theta}$ and $\bar{\theta}$ respectively denote the minimum and maximum values of J.

A DRP is characterized by a pair of functions (λ, p), where for each $s \in J$, $\lambda(s)$ and $p(s)$ are respectively the probability with which trade occurs and the price at which it occurs if the seller announces that the value of θ is s. A DRP is incentive-compatible if and only if

$$\forall \theta \in J, \quad \lambda(\theta)[p(\theta) - c(\theta)] \geq \lambda(s)[p(s) - c(\theta)], \quad \forall s \in J. \tag{9.7}$$

Inequality 9.7 states that for any possible true (realized) value of θ, the expected payoff to the θ-type seller by announcing the truth is greater than or equal to her expected payoff by telling a lie.[7]

An incentive-compatible DRP is individually-rational if and only if

$$\forall \theta \in J, \quad \lambda(\theta)[p(\theta) - c(\theta)] \geq 0, \quad \text{and} \tag{9.8}$$

$$E_\theta\Big[\lambda(\theta)[v(\theta) - p(\theta)]\Big] \geq 0. \tag{9.9}$$

Inequalities 9.8 and 9.9 respectively state that the expected payoffs to the θ-type seller and the buyer in an incentive-compatible DRP are greater than or equal to their respective payoffs from disagreement (which equal zero). Since (by assumption) $v(\theta) \geq c(\theta)$ for all θ, a DRP is *ex-post* efficient if and only if

$$\forall \theta \in J, \quad \lambda(\theta) = 1. \tag{9.10}$$

The following claim is a generalization of Claim 9.1.

[7] The seller is said to be of 'θ-type' if the realization of the random draw from G is θ.

Claim 9.2. *If $v^e < c(\bar{\theta})$, where v^e is the buyer's expected reservation value — that is, $v^e = E_\theta[v(\theta)]$ — then there does not exist an incentive-compatible and individually-rational DRP that is ex-post efficient.*

Proof. Suppose, to the contrary, that there exists a DRP that satisfies (9.7)–(9.10). Substituting (9.10) into (9.7) implies that $p(\theta)$ is constant for all $\theta \in J$. Letting $p = p(\theta)$ for all $\theta \in J$, it follows using (9.8) that $p \geq c(\bar{\theta})$. Hence, using (9.9) it follows that $v^e \geq c(\bar{\theta})$, which contradicts the hypothesis. □

It follows from the Revelation Principle that if $v^e < c(\bar{\theta})$, then for any bargaining procedure the induced bargaining game does not have an *ex-post* efficient BNE. Therefore, if $v^e < c(\bar{\theta})$, then the bargaining outcome cannot be *ex-post* efficient. On the other hand, if $v^e \geq c(\bar{\theta})$, then there exists a bargaining procedure such that the induced bargaining game has an *ex-post* efficient BNE.[8]

Remark 9.1 (The Buyer has Private Information). Now consider the case in which it is the buyer (and not the seller) who has private information about θ. Thus, the realization of the random draw from the probability distribution G is only revealed to the buyer. It is straightforward to appropriately modify the above analysis and show that the bargaining outcome can be *ex-post* efficient if and only if $c^e \leq v(\underline{\theta})$, where c^e is the seller's expected reservation value — that is, $c^e = E_\theta[c(\theta)]$.

Hence, I have established the following proposition.

Proposition 9.2 (One-Sided Uncertainty with Correlated Values).
(a) When the seller has private information about θ the bargaining outcome can be ex-post efficient if and only if the buyer's expected reservation value $v^e \geq c(\bar{\theta})$, the seller's maximum possible reservation value.
(b) When the buyer has private information about θ the bargaining outcome can be ex-post efficient if and only if the seller's expected reservation value $c^e \leq v(\underline{\theta})$, the buyer's minimum possible reservation value.

[8] Consider the following bargaining procedure. The seller makes an offer to the buyer. If she accepts the offer, then agreement is struck and the game ends. But if she rejects the offer, then the game ends with no agreement. If $v^e \geq c(\bar{\theta})$, then the following pair of strategies is a BNE: $p^*(\theta) = c(\bar{\theta})$ (where $p^*(\theta)$ is the seller's price offer when the realization of the random draw from G is θ), the buyer accepts the price $p = c(\bar{\theta})$ and rejects any price $p \neq c(\bar{\theta})$.

An Example in which Trade Never Occurs

Suppose that J is equal to the closed interval $[0, 1]$, G is a uniform distribution (i.e., $G(\theta) = \theta$), $v(\theta) = 3\theta$ and $c(\theta) = 2\theta$. Assume that the seller has private information about θ. Since $v^e = 1.5$ and $c(1) = 2$, it follows from Proposition 9.2(a) that the bargaining outcome cannot be *ex-post* efficient. In fact, it is shown below that in any incentive-compatible and individually-rational DRP, $\lambda(\theta) = 0$ for all $\theta \in [0, 1]$. This striking result implies (by appealing to the Revelation Principle) that for any bargaining procedure and any BNE of the induced bargaining game trade occurs with probability zero.

Fix an arbitrary incentive-compatible and individually-rational DRP. Let U_B and $U_S(\theta)$ respectively denote the expected payoffs (in the induced bargaining game) to the buyer and the θ-type seller. Inequality 9.7 implies that for each $\theta \in [0, 1]$

$$U_S(\theta) \equiv \lambda(\theta)[p(\theta) - 2\theta] = \max_{s \in [0,1]} \lambda(s)[p(s) - 2\theta].$$

From the *Envelope Theorem*, it follows that $U_S(.)$ is differentiable almost everywhere with derivative $U'_S(\theta) = -2\lambda(\theta)$. This implies that

$$\int_0^1 \theta dU_S(\theta) = -2 \int_0^1 \theta\lambda(\theta)d\theta. \tag{9.11}$$

After integrating by parts the LHS of equation 9.11, and then simplifying, it follows that

$$\int_0^1 U_S(\theta)d\theta = U_S(1) + 2\int_0^1 \theta\lambda(\theta)d\theta. \tag{9.12}$$

Now consider the expected payoff U_B to the buyer, which is the LHS of inequality 9.9. After substituting for $\lambda(\theta)p(\theta)$ using the expression for $U_S(\theta)$, it follows that

$$U_B = \int_0^1 [\theta\lambda(\theta) - U_S(\theta)]d\theta,$$

which (using (9.12)) implies that

$$U_B = -U_S(1) - \int_0^1 \theta\lambda(\theta)d\theta.$$

Hence, it follows from inequalities 9.8 and 9.9 that

$$\int_0^1 \theta\lambda(\theta)d\theta \leq 0,$$

which implies that $\lambda(\theta) = 0$ for all $\theta \in [0, 1]$.

9.3 Applications

9.3.1 Efficient Wage Agreements

Consider a firm whose workforce is represented by a union. The firm and the union are bargaining over the wage rate on the assumption that there will be no firing and hiring. In order to simplify the notation, normalize the mass of workers employed at this firm to unity. Assuming that the union's payoff is the same as a worker's payoff, if agreement is reached on wage rate w, then the (average) payoffs to the firm and the union are respectively $R - w$ and w, where R ($R > 0$) is the value of the (average) output generated by the firm's workforce. If the players do not reach agreement on a wage rate, then the entire workforce goes on indefinite strike. In this eventuality the firm shuts down and obtains a payoff of zero, while each worker has recourse to the union's strike fund and obtains an average payoff of α, where $\alpha > 0$. The bargaining outcome is *ex-post* efficient if and only if when $R \geq \alpha$ the players reach a wage agreement, and when $R < \alpha$ the union goes on indefinite strike and the firm shuts down.

It is helpful to normalize the union's payoffs so that its average payoff from disagreement is zero. This implies that its (normalized) average payoff from agreement on wage rate w is $w - \alpha$. Indeed, the firm and the union are bargaining over the price at which the union will sell a fixed amount of labour to the firm, where the firm's and the union's reservation values are respectively R and α.

It follows from Proposition 9.1 that if either the firm has private information about R, or the union has private information about α (but not both), then the bargaining outcome can be *ex-post* efficient. That is, there exists a bargaining procedure whose induced bargaining game has a Bayesian Nash equilibrium with the following property: when $R \geq \alpha$ a wage agreement is reached, and when $R < \alpha$ the union goes on indefinite strike. It should be noted that it is not unreasonable that the value of the (average) output generated by the entire workforce is the firm's private information, and that the magnitude of the union's strike fund is its private information.[9]

9.3.2 Litigation or Out-of-Court Settlement

Individual D has injured individual P, and they are bargaining over the amount of monetary compensation that D will give P. The individuals can either reach an agreement (and thus settle out-of-court), or litigate. If they agree to settle out-of-court with individual D — the defendant — paying individual P — the plantiff — an amount p ($p \geq 0$), then the payoffs to the defendant and the plantiff are respectively $-p$ and p. If, on the other hand, they litigate, then with probability γ ($0 < \gamma < 1$) the court will find the defendant guilty of the crime, in which case she will be required to pay an amount x ($x > 0$) to the plantiff. However, with probability $1 - \gamma$ the defendant will not be found guilty, in which case she pays nothing to the plantiff. Litigation will cost each individual an amount f ($f > 0$).[10] Hence, the (expected) payoffs from litigation to the defendant and the plantiff are respectively $-\gamma x - f$ and $\gamma x - f$.

It is helpful to normalize the players' payoffs so that each player's payoff from litigation is zero. This implies that the defendant's and the plantiff's (normalized) payoffs from an out-of-court settlement on price p are respectively $v - p$ and $p - c$, where $v = \gamma x + f$ and $c = \gamma x - f$. Furthermore, each player's payoff from disagreement — that is, from litigation — is zero. Indeed, the defendant and the plantiff are bargaining over the price at which the plantiff will sell her claim to the lawsuit, where the defendant's and the plantiff's reservation values are respectively v and c. Notice that their reservation values are correlated.

I apply Proposition 9.2. First, consider the case in which the plantiff (the seller) has private information about x — which is not an unreasonable assumption, because it is possible that the plantiff only knows the exact extent of her injury. In the current bargaining situation the condition $v^e \geq c(\bar{x})$ can be written as $f \geq \gamma(\bar{x} - x^e)/2$, where x^e is the expected value of x. Hence, the bargaining outcome can be *ex-post* efficient if and only

[9] In Section 9.5.1 it is shown that if the firm has private information about R *and* the union has private information about α, then the bargaining outcome cannot be *ex-post* efficient.

[10] This is partly because litigation involves hiring a lawyer.

if the cost of litigation to each player $f \geq \gamma(\bar{x} - x^e)/2$. In particular, if $f < \gamma(\bar{x} - x^e)/2$, then for any bargaining procedure and any BNE of the induced (pre-trial) bargaining game with a strictly positive probability the plantiff and the defendant will proceed to litigation.

Now consider the case in which the value of x is known to both players, but the defendant (the buyer) has private information about γ — which is not an unreasonable assumption, because it is possible that the defendant only knows the exact extent to which her crime is provable in a court of law.[11] In the current bargaining situation the condition $c^e \leq v(\underline{\gamma})$ can be written as $f \geq x(\gamma^e - \underline{\gamma})/2$, where γ^e is the expected value of γ. Hence, the bargaining outcome can be *ex-post* efficient if and only if the cost of litigation to each player $f \geq x(\gamma^e - \underline{\gamma})/2$. In particular, if $f < x(\gamma^e - \underline{\gamma})/2$, then for any bargaining procedure and any BNE of the induced (pre-trial) bargaining game with a strictly positive probability the plantiff and the defendant will proceed to litigation.

The Effect of a Fee-Shifting Rule

In the analysis above it is (implicitly) assumed that each player bears her cost of litigation. This is known as the American rule, because it is typical in the USA. In England, on the other hand, it is typically the case that the loser bears the winner's cost of litigation — hence, this is called the English rule. I now consider the normative question in the context of the English rule.

If the players proceed to litigation, then with probability γ the court will find the defendant guilty of the crime. Hence, with probability γ the plantiff wins the lawsuit, while with probability $1 - \gamma$ the defendant is the winner. This implies that the (expected) payoffs from litigation to the defendant and the plantiff are respectively $-\gamma x - 2\gamma f$ and $\gamma x - 2(1-\gamma)f$. Notice that these disagreement payoffs differ from those under the American rule. As above, normalize the players' payoffs so that each player's payoff from litigation is zero. This implies that the defendant's and the plantiff's (normalized) payoffs from an out-of-court settlement on price p are respectively $v - p$ and $p - c$, where $v = \gamma x + 2\gamma f$ and $c = \gamma x - 2(1-\gamma)f$.

[11] In Section 9.5.1, I analyse the (pre-trial) bargaining situation under the assumption that the plantiff has private information about x *and* the defendant has private information about γ.

I apply Proposition 9.2. First, consider the case in which the plantiff has private information about x. It is straightforward to show that the conclusion is the same as under the American rule. Now consider the case in which the value of x is known to both players, but the defendant (the buyer) has private information about γ. In this case it follows that the bargaining outcome can be *ex-post* efficient if and only if the cost of litigation to each player $f \geq x(\gamma^e - \underline{\gamma})/2(1 - \gamma^e + \underline{\gamma})$. Since $\gamma^e > \underline{\gamma}$, it follows that if

$$\frac{x(\gamma^e - \underline{\gamma})}{2} < f < \frac{x(\gamma^e - \underline{\gamma})}{2(1 - \gamma^e + \underline{\gamma})},$$

then under the American rule the bargaining outcome can be *ex-post* efficient, while under the English rule it cannot be *ex-post* efficient.

A main message contained in the results obtained above is as follows. If and only if the probability with which the plantiff wins at trial is the defendant's private information, then the disputants are more likely to proceed to litigation under the English rule than under the American Rule.

9.4 Efficiency under Two-Sided Uncertainty

This section extends the analysis of Section 9.2 to the case when each player has some private information. As in Section 9.2, I study the bargaining situation in which player S owns (or, can produce) an indivisible object that player B wants to buy. If agreement is reached to trade at price p ($p \geq 0$), then the payoffs to the seller (player S) and the buyer (player B) are respectively $p - c$ and $v - p$, where c denotes the seller's reservation value (or, cost of production) and v denotes the buyer's reservation value (or, the maximum price at which she is willing to buy). If the players do not reach an agreement to trade, then each player's payoff is zero.

Section 9.4.1 studies the case in which the players' reservation values are independent of each other, and each player's reservation value is her private information. Section 9.4.2 studies the case in which the players' reservation values are correlated, and each player has some relevant private information.

As in Section 9.2, my objective is to address the normative question of whether or not the bargaining outcome can be *ex-post* efficient.[12] As in

[12]The outcome of this bargaining situation is *ex-post* efficient if and only if when $v \geq c$

Section 9.2.2, the analysis involves studying the set of incentive-compatible and individually-rational direct revelation procedures.

9.4.1 The Case of Private Values

In this section the players' reservation values are independent of each other, the seller's reservation value is her private information, and the buyer's reservation value is her private information. This asymmetry in information is modelled as follows. The buyer's reservation value is a random draw from a probability distribution F_B, and the seller's reservation value is an independent and random draw from a probability distribution F_S. The buyer knows the realization of the draw from F_B, but the seller does not: she only knows that the buyer's reservation value is an independent and random draw from F_B. Symmetrically, the seller knows the realization of the draw from F_S, but the buyer does not: she only knows that the seller's reservation value is an independent and random draw from F_S.

Letting I_i $(i = B, S)$ denote the support of F_i, denote the minimum and maximum values of I_B respectively by $\underline{v}$ and $\bar{v}$, and the minimum and maximum values of I_S respectively by $\underline{c}$ and $\bar{c}$.

In the context of the bargaining situation under consideration, a direct revelation procedure (DRP) is characterized by a pair of functions (λ, p). In a DRP the seller and the buyer simultaneously announce their respective reservation values. If the seller's announced value is c $(c \in I_S)$ and the buyer's announced value is v $(v \in I_B)$, then with probability $\lambda(c, v)$ trade occurs at price $p(c, v)$, and with probability $1 - \lambda(c, v)$ trade does not occur.

A DRP is *incentive-compatible* if and only if the following inequalities are satisfied

$$\forall c, c' \in I_S, \quad U_S(c) \equiv E_v\Big[\lambda(c, v)[p(c, v) - c]\Big] \geq E_v\Big[\lambda(c', v)[p(c', v) - c]\Big] \quad (9.13)$$

$$\forall v, v' \in I_B, \quad U_B(v) \equiv E_c\Big[\lambda(c, v)[v - p(c, v)]\Big] \geq E_c\Big[\lambda(c, v')[v - p(c, v')]\Big]. \quad (9.14)$$

the players reach an agreement to trade, and when $v < c$ the players do not reach an agreement to trade.

An incentive-compatible DRP is *individually-rational* if and only if

$$\forall c \in I_S, \quad U_S(c) \geq 0 \quad \text{and} \tag{9.15}$$

$$\forall v \in I_B, \quad U_B(v) \geq 0. \tag{9.16}$$

A DRP is *ex-post efficient* if and only if the buyer and the seller trade when it is mutually beneficial to do so, but not otherwise. That is, if and only if for each $c \in I_S$ and $v \in I_B$

$$\lambda(c, v) = \begin{cases} 1 & \text{if } v \geq c \\ 0 & \text{if } v < c. \end{cases} \tag{9.17}$$

In Proposition 9.3(a) below it is shown that if $\underline{v} \geq \bar{c}$ — which implies that gains from trade exist with probability one — then the bargaining outcome can be *ex-post* efficient. In contrast, in Proposition 9.3(b) it is shown that under some conditions on the distributions — which imply that there is uncertainty over whether or not gains from trade exist — the bargaining outcome cannot be *ex-post* efficient.

Proposition 9.3 (Two-Sided Uncertainty with Private Values). *(a) If $\underline{v} \geq \bar{c}$, then the bargaining outcome can be ex-post efficient.*
(b) If $I_B = [\underline{v}, \bar{v}]$, $I_S = [\underline{c}, \bar{c}]$, F_i $(i = B, S)$ has a continuous and strictly positive density, and the interiors of the intervals I_B and I_S have a non-empty intersection, then the bargaining outcome cannot be ex-post efficient.

Proof. I first prove of part (a). Consider the following bargaining procedure. The seller makes an offer to the buyer. If she accepts the offer, then agreement is struck and the game ends. But if she rejects the offer, then the game ends with no agreement. Since $\underline{v} \geq \bar{c}$, the following pair of strategies is a BNE: $p^*(c) = \bar{c}$ (where $p^*(c)$ is the seller's price offer when her reservation value is c), the buyer accepts the price $p = \bar{c}$ and rejects any price $p \neq \bar{c}$ whatever is her reservation value. The desired conclusion follows immediately, because this BNE is *ex-post* efficient.

Since the proof of part (b) is a bit technical, I omit it, and instead refer the interested reader to Myerson and Satterthwaite (1983) — the authors of this result. However, as I now show, it is straightforward to establish part (b) under the following specific assumptions: $I_B = I_S = [0, 1]$ and

F_i $(i = B, S)$ is uniform (i.e., $F_i(x) = x$ for $x \in [0,1]$).[13] By the Revelation Principle it suffices to show that there does not exist an incentive-compatible and individually-rational DRP that is *ex-post* efficient. Suppose, to the contrary, that there exists a DRP that satisfies (9.13)–(9.17). Inequality 9.13 implies (using the *Envelope Theorem*) that $U_S(c)$ is differentiable almost everywhere with derivative $U_S'(c) = -E_v[\lambda(c,v)]$. Since (9.17) implies that $E_v[\lambda(c,v)] = 1 - c$, it follows that the seller's (unconditional) expected payoff $U_S^e = U_S(1) + 1/6$. By a symmetric argument it follows that the buyer's (unconditional) expected payoff $U_B^e = U_B(0) + 1/6$. However, by definition, the sum of the players' (unconditional) expected payoffs is $E_{v,c}[(v-c)\lambda(c,v)]$. Substituting (9.17) into this expression, and integrating, it follows that $E_{v,c}[(v-c)\lambda(c,v)] = 1/6$. This therefore implies that $U_S(1) + U_B(0) = -1/6$, which contradicts (9.15)–(9.16). □

9.4.2 The Case of Correlated Values

In this section the player's reservation values are correlated, and each player has some relevant private information. This assumption is modelled as follows. There are two parameters θ and ρ — both of which are real numbers — that determine both players' reservation values, and, furthermore, the value of θ is the private information of the seller and the value of ρ is the private information of the buyer. The asymmetry in information is modelled as follows. The values of θ and ρ are independent and random draws from the probability distributions G and H, respectively. The seller knows the realization of the draw from G, but the buyer does not: she only knows that the value of θ is an independent and random draw from G. Symmetrically, the buyer knows the realization of the draw from H, but the seller does not: she only knows that the value of ρ is an independent and random draw from H.

Letting J and K respectively denote the supports of G and H, denote the minimum and maximum values of J respectively by $\underline{\theta}$ and $\bar{\theta}$, and the minimum and maximum values of K respectively by $\underline{\rho}$ and $\bar{\rho}$.

Both the seller's reservation value — denoted by $c(\theta, \rho)$ — and the buyer's reservation value — denoted by $v(\theta, \rho)$ — are strictly increasing

[13] The conceptual argument in the proof of the general case in Myerson and Satterthwaite (1983) is similar to that in the proof to follow in this special case.

in θ and in ρ. Furthermore, assume that for any θ and ρ, $v(\theta, \rho) \geq c(\theta, \rho)$.

In the context of the bargaining situation under consideration, a direct revelation procedure (DRP) is characterized by a pair of functions (λ, p). In a DRP the seller announces the value of θ and simultaneously the buyer announces the value of ρ. If the seller's announced value is θ' $(\theta' \in J)$ and the buyer's announced value is ρ' $(\rho' \in K)$, then with probability $\lambda(\theta', \rho')$ trade occurs at price $p(\theta', \rho')$, and with probability $1 - \lambda(\theta', \rho')$ trade does not occur.

A DRP is *incentive-compatible* if and only if the following inequalities are satisfied

$$\forall \theta, \theta' \in J, \quad U_S(\theta) \equiv E_\rho\Big[\lambda(\theta, \rho)[p(\theta, \rho) - c(\theta, \rho)]\Big] \geq E_\rho\Big[\lambda(\theta', \rho)[p(\theta', \rho) - c(\theta, \rho)]\Big] \tag{9.18}$$

$$\forall \rho, \rho' \in K, \quad U_B(\rho) \equiv E_\theta\Big[\lambda(\theta, \rho)[v(\theta, \rho) - p(\theta, \rho)]\Big] \geq E_\theta\Big[\lambda(\theta, \rho')[v(\theta, \rho) - p(\theta, \rho')]\Big]. \tag{9.19}$$

An incentive-compatible DRP is *individually-rational* if and only if

$$\forall \theta \in J, \quad U_S(\theta) \geq 0 \quad \text{and} \tag{9.20}$$

$$\forall \rho \in K, \quad U_B(\rho) \geq 0. \tag{9.21}$$

A DRP is *ex-post efficient* if and only if

$$\forall \theta \in J \text{ and } \forall \rho \in K, \quad \lambda(\theta, \rho) = 1. \tag{9.22}$$

The following proposition addresses the normative question of whether or not the bargaining outcome can be *ex-post* efficient.

Proposition 9.4 (Two-Sided Uncertainty with Correlated Values). *The bargaining outcome can be ex-post efficient if and only if $v^e(\underline{\rho}) \geq c^e(\bar{\theta})$, where $v^e(\underline{\rho}) = E_\theta[v(\theta, \underline{\rho})]$ and $c^e(\bar{\theta}) = E_\rho[c(\bar{\theta}, \rho)]$.*

Proof. I first establish *sufficiency*. Consider the following bargaining procedure. The buyer makes a price offer to the seller. If she accepts the offer, then agreement is struck and the game ends. But if she rejects the offer, then the game ends with no agreement. If $v^e(\underline{\rho}) \geq c^e(\bar{\theta})$, then the following pair of strategies is a BNE: for any value of ρ the buyer offers the price $p = c^e(\bar{\theta})$, and for any value of θ the seller accepts the price $p = c^e(\bar{\theta})$ and rejects any price $p \neq c^e(\bar{\theta})$. The desired conclusion follows immediately, because this BNE is *ex-post* efficient. I now establish *necessity*. By the Revelation Principle it suffices to show that if $v^e(\underline{\rho}) < c^e(\bar{\theta})$, then there does not exist a DRP that satisfies (9.18)–(9.22). Suppose, to the contrary, that there exists such a DRP. After substituting the *ex-post* efficiency condition (9.22) into the seller's incentive-compatibility condition (9.18), it follows that the expectation of $p(\theta, \rho)$ with respect to ρ is independent of θ. Let it be denoted by p^e_S. Similarly, after substituting the *ex-post* efficiency condition (9.22) into the buyer's incentive-compatibility condition (9.19), it follows that the expectation of $p(\theta, \rho)$ with respect to θ is independent of ρ. Let it be denoted by p^e_B. After substituting (9.22) into (9.20) it thus follows from the seller's individual-rationality condition (9.20) that for any $\theta \in J$, $p^e_S \geq E_\rho[c(\theta, \rho)]$. Symmetrically, after substituting (9.22) into (9.21) it follows from the buyer's individual-rationality condition (9.21) that for any $\rho \in K$, $E_\theta[v(\theta, \rho)] \geq p^e_B$. This implies that $p^e \geq c^e(\bar{\theta})$ and $v^e(\underline{\rho}) \geq p^e$, where p^e is the expectation of $p(\theta, \rho)$ with respect to θ and ρ. Consequently, $v^e(\underline{\rho}) \geq c^e(\bar{\theta})$, which contradicts the hypothesis. □

9.5 Applications

9.5.1 Indefinite Strikes

Consider the bargaining situation between the firm and its union as laid out in Section 9.3.1, but with the assumption that the firm has private information about R *and* the union has private information about α. Assume that R and α are independent and random draws from two continuous probability distributions with strictly positive densities, where R and α respectively take values from the closed intervals $[\underline{R}, \bar{R}]$ and $[\underline{\alpha}, \bar{\alpha}]$.

It follows from Proposition 9.3(a) that if $\underline{R} \geq \bar{\alpha}$, then the bargaining outcome can be *ex-post* efficient. Thus, if with probability one it is mutually beneficial for the union to sell its labour to the firm, then there exists a bargaining procedure whose induced bargaining game has a BNE with the following property: with probability one a wage agreement is reached.